THE LOAD PLANNING PROBLEM FOR LESS-THAN-TRUCKLOAD MOTOR CARRIERS AND A SOLUTION APPROACH

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ABSTRUCT
The main stress in the load planning problem for less-than-truckload (LTL) motor carriers falls on determining how to consolidate freight on small-lot consignment over a load planning network including break-bulk terminals. The goal of this problem is to minimize the total line-haul cost under the condition that the minimum frequency of delivery per week between a pair of terminals must satisfy a given service level. In this study, we propose a load planning model and new algorithm using a Lagrangian relaxation method. Numerical experiments are presented to evaluate the effectiveness of our Lagrangian relaxation method.

INTRODUCTION
Progress of the supply chain management and the current trend toward deregulation of the Japanese trucking industry places the freight motor carriers in a highly competitive environment. As a result of that, the carriers need to consider strategies and tactics that satisfy both cost minimization and a definite level of service quality. In general, a less-than-truckload motor carrier hauls shipments in the range of 50kg to 5000kg. Since a standard trailer can hold 10-ton to 30-ton of shipment, it is necessary for the LTL motor carriers to consolidate the freight to make the best use of trailers. The freight originating at an end-of-line is loaded onto a line-haul truck, which carries it to a break-bulk terminal. At this terminal, the freight is unloaded, sorted and reloaded onto a trailer, which carries it to another terminal. One of the main problems faced by LTL motor carriers is to determine how freight should be routed over the network. This problem is called the load planning problem for LTL motor carriers. It can be formulated as a huge mixed integer optimization problem.

Previous research on this problem is limited. Powell (1986) and Powell-Sheffi (1989) propose heuristic approaches using add-drop local search methods. Crainic-Roy (1992) describes a set-covering formulation and a solution method for the load planning problem. Powell-Delorme (1989) presents so-called NETPLAN, and Powell-Koskosidis (1992) uses a gradient-based local search method and the Lagrangian heuristic approach with a relaxation of minimum service level constraints. Hoppe et al.(1999) proposes a heuristic approach using the labeling algorithm and add-drop local search methods. Crainic (1999) surveys wide variety of freight transportation planning problems.

The principal concern in the load planning problem is about determining how to consolidate freight on small-lot consignment over a load planning network including break-bulk terminals in order to minimize the total line-haul cost. This problem is approached as a two-tiered problem: 1) between which pairs of terminals should direct service be offered, 2) given a set of direct services, how should the freight be routed over the network. This problem is formulated as follows: 1) line-haul costs between terminals should be minimized, 2) the minimum frequency of delivery per week between a pair of terminals must satisfy a given service level, 3) the paths from all origin terminals into a destination terminal form a tree, which reflects that the freight at a terminal with same destination should be loaded onto a

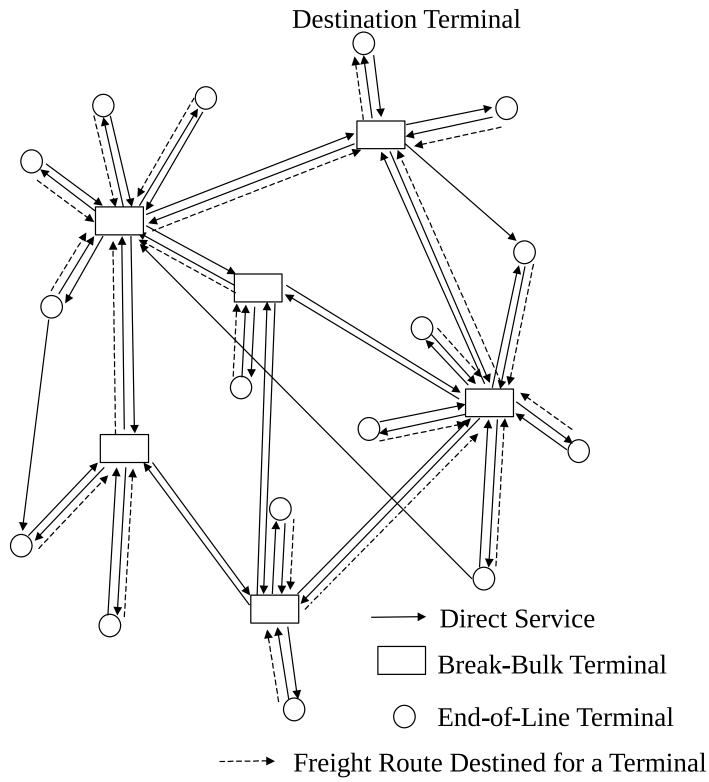

Figure 1. Illustrative Load Planning Network

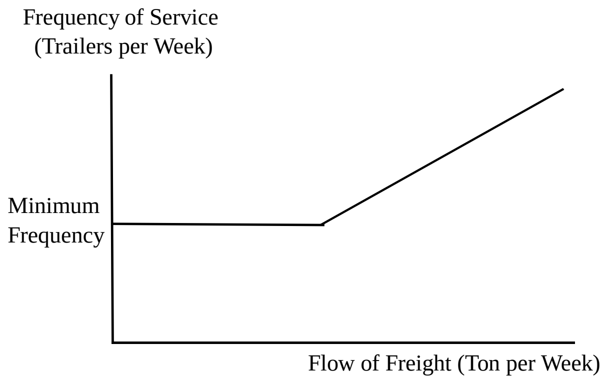

Figure 2. Frequency of Direct Service between a Pair of Terminals

truck heading for one terminal. Figure 1 illustrates an example of the load planning network. Figure 2 illustrates frequency of service between a pair of terminals. In this study, we propose a load planning model and new algorithm using a Lagrangian relaxation method.

FORMULATION FOR THE LOAD PLANNING PROBLEM

The load planning problem can be formulated as a mixed integer programming problem. We use the following notation for our model. N is the set of nodes, which consists of end-of-line and break-bulk terminals. A is the set of all potential links for direct services in the load planning network, $A \subseteq N \times N$. f_{ij} is the minimum frequency of trailers per week on the link (i,j) from node i to j, $(i,j) \in A$. e_{ij} is the load capacity of a trailer on the link (i,j). a_{ij} is the line-haul cost per trailer on the link (i,j). q^{od} is the total LTL freight demand of commodity (o,d) originating at terminal o and destined for terminal d per week, $o \in N$, $d \in N$. δ_n^{od} is a constant, which equals to 1 if node n is destination d, -1 if node n is origin o and 0 otherwise, $n,o,d \in N$. z_{ij} is the total line-haul cost on the link (i,j). x_{ij} is the total freight flow on the link (i,j). x_{ij}^{od} is the binary decision variable, which equals to 1 if the freight flow of commodity (o,d) is routed on the link (i,j) and 0 if not. y_{ij} is the binary decision variable, which equals to 1 if a direct service is being offered on the link (i,j) and 0 if not. y_{ij}^{d} is the binary decision variable, which equals to 1 if the freight flow destined for terminal d is routed on the link(i,j) and 0 if not.

The load planning problem can be stated as follows.

$$(LTL) \quad minimize \quad \sum_{(i,j)\in A} z_{ij} y_{ij} \qquad (1)$$

$$subject\ to \quad \sum_{i\in N} x_{in}^{od} - \sum_{j\in N} x_{nj}^{od} = \delta_n^{od} \quad n \in N, o \in N, d \in N \qquad (2)$$

$$x_{ij} = \sum_{o\in N} \sum_{d\in N} q^{od} x_{ij}^{od} \quad (i,j) \in A \qquad (3)$$

$$x_{ij}^{od} \le y_{ij}^{d} \quad (i,j) \in A, o \in N, d \in N \qquad (4) \qquad y_{ij}^{d} \le y_{ij} \quad (i,j) \in A, d \in N \qquad (5)$$

$$\sum_{j\in N} y_{ij}^{d} \le 1 \quad i \in N \setminus \{d\}, d \in N \qquad (6) \qquad \sum_{j\in N} y_{dj}^{d} = 0 \quad d \in N \qquad (7)$$

$$z_{ij} = \begin{cases} a_{ij}x_{ij}/e_{ij} & \text{if } x_{ij} \ge f_{ij}e_{ij} \\ a_{ij}f_{ij} & \text{if } x_{ij} < f_{ij}e_{ij} \end{cases} \quad (i,j)\in A \quad (8) \qquad x_{ij}^{od} \in \{0,1\} \quad (i,j)\in A, o\in N, d\in N \quad (9)$$

$$y_{ij} \in \{0,1\} \quad (i,j)\in A \qquad (10) \qquad y_{ij}^{d} \in \{0,1\} \quad (i,j)\in A, d\in N \qquad (11)$$

The objective function (1) is the total line-haul cost and should be minimized. Constraint (2) expresses the standard flow conservation. Constraint (3) shows the relationship between the total freight flow and the demands of commodity (o,d). Constraint (4) states that if the flow destined for d is not routed on the link(i,j), then the flow of every commodity (o,d) on the link (i,j) must be zero. Constraint (5) states that if the direct service on the link (i,j) is not being offered, then the flow destined for d must not be routed on the link (i,j). Constraints (6) and (7) insure that the paths from all origin terminals into a destination terminal form a tree. Constraint (8) states that the minimum frequency of deliveries between a pair of terminals must satisfy a given service level. Constraints (9), (10) and (11) are the binary requirements.

A LAGRANGIAN RELAXATION PROBLEM

The Lagrangian relaxation is one of general solution strategies for solving mathematical programming problems that permit us to decompose problems to exploit their special structure. When we use vectors of the Lagrange multipliers $\boldsymbol{v}=\{v_i^{od}\}$ relative to constraint (2) and $\boldsymbol{w}=\{w_i^{d}\}$ relative to (6) and (7), and add them to the objective function (1), the following Lagrangian relaxation problem LG can be formed.

$$(LG) \quad \textit{minimize} \quad \sum_{(i,j)\in A} z_{ij}y_{ij} + \sum_{(i,j)\in A}\sum_{d\in N}\left\{\sum_{o\in N}\left(v_i^{od} - v_j^{od}\right)x_{ij}^{od} + w_i^{d}y_{ij}^{d}\right\}$$
$$+\sum_{d\in N}\sum_{o\in N}\left(v_d^{od} - v_o^{od}\right) - \sum_{d\in N}\sum_{i\in N\setminus\{d\}} w_i^{d} \qquad (12)$$

subject to (3)-(5) *and* (8)-(11)

where $w_i^d \ge 0$, $i \in N \setminus \{d\}$, $d \in N$.

Given the Lagrange multipliers $\boldsymbol{v}$ and $\boldsymbol{w}$, we can deal with the third and the fourth terms of the objective function (12) as constant terms. LG can be decomposed into following subproblem LG_{ij} for each link (i,j).

$$(LG_{ij}) \quad \textit{minimize} \quad z_{ij}y_{ij} + \sum_{d\in N}\left\{\sum_{o\in N}\left(v_i^{od} - v_j^{od}\right)x_{ij}^{od} + w_i^{d}y_{ij}^{d}\right\} \qquad (13)$$

$$\textit{subject to} \quad x_{ij}^{od} \le y_{ij}^{d} \quad o\in N, d\in N \quad (14) \qquad y_{ij}^{d} \le y_{ij} \quad d\in N \quad (15)$$

$$z_{ij} = \begin{cases} \sum_{o\in N}\sum_{d\in N} a_{ij}q^{od}x_{ij}^{od}/e_{ij} & \text{if } \sum_{o\in N}\sum_{d\in N} q^{od}x_{ij}^{od} \ge f_{ij}e_{ij} \\ a_{ij}f_{ij} & \text{if } \sum_{o\in N}\sum_{d\in N} q^{od}x_{ij}^{od} < f_{ij}e_{ij} \end{cases} \qquad (16)$$

$$x_{ij}^{od} \in \{0,1\} \quad o\in N, d\in N \quad (17) \qquad y_{ij} \in \{0,1\} \quad (18)$$

$$y_{ij}^{d} \in \{0,1\} \quad d\in N \quad (19)$$

Furthermore, LG_{ij} can be decomposed into following two subproblems, LG_{ij}^{1} and LG_{ij}^{2}.

$$(LG_{ij}^{1}) \quad \textit{minimize} \quad \sum_{d\in N}\left\{\sum_{o\in N}\left(a_{ij}q^{od}/e_{ij}y_{ij} + v_i^{od} - v_j^{od}\right)x_{ij}^{od} + w_i^{d}y_{ij}^{d}\right\} \qquad (20)$$

$$\textit{subject to} \quad \sum_{o\in N}\sum_{d\in N} q^{od}x_{ij}^{od} \ge f_{ij}e_{ij} \qquad (21)$$

(14),(15) *and* (17)-(19)

$$(LG_{ij}^{2}) \quad \textit{minimize} \quad a_{ij}f_{ij} + \sum_{d\in N}\left\{\sum_{o\in N}\left(v_i^{od} - v_j^{od}\right)x_{ij}^{od} + w_i^{d}y_{ij}^{d}\right\} \qquad (22)$$

$$\textit{subject to} \qquad \sum_{o\in N}\sum_{d\in N} q^{od} x_{ij}^{od} < f_{ij} e_{ij} \tag{23}$$
$$(14),(15) \textit{ and } (17)\text{-}(19)$$

When we give the Lagrange multipliers ***v*** and ***w***, and solve the Lagrangian relaxation problem *LG* or further a relaxation problem optimally, a lower bound for *LTL* can be obtained.

A OPTIMAL SOLUTION FOR A LAGRANGIAN RELAXATION PROBLEM

At first, we assume that the Lagrange multipliers ***v*** are given and let ***w***=**0**. Then LG_{ij}^{1} can be rewritten as a simple problem.

$$\textit{minimize} \qquad \sum_{d\in N}\sum_{o\in N}\left(a_{ij} q^{od}/e_{ij} y_{ij} + v_i^{od} - v_j^{od}\right) x_{ij}^{od} \tag{24}$$
$$\textit{subject to} \qquad x_{ij}^{od} \le y_{ij} \quad o\in N, d\in N \tag{25}$$
$$(17),\ (18) \textit{ and } (21)$$

We decompose this problem into two subproblems in the case of y_{ij}=0 and y_{ij}=1. Obviously, when y_{ij}=0, the optimal solution is $x_{ij}^{od}=0(\forall\, o,d\in N)$ and the optimal value of equation (24) is 0. When y_{ij}=1, this problem can be rewritten as the following 0-1 knapsack problem LG_{ij}^{11}.

$$(LG_{ij}^{11}) \qquad \pi_{ij}^{1} = \textit{minimize} \ \sum_{d\in N}\sum_{o\in N}\left(a_{ij} q^{od}/e_{ij} + v_i^{od} - v_j^{od}\right) x_{ij}^{od} \tag{26}$$
$$\textit{subject to} \quad (17) \textit{ and } (21)$$

This problem relaxed 0-1 conditions turn out to be the continuous knapsack problem and can be simply solved by sorting, and then a lower bound and the relaxation solution for LG_{ij}^{11} are easily obtained. Accordingly the lower bound for LG_{ij}^{1} is $min\{0, \pi_{ij}^{1}\}$, which is the minimum value of the optimum in the case of y_{ij}=0 and y_{ij}=1.

As with LG_{ij}^{1}, LG_{ij}^{2} can be decomposed into two cases of y_{ij}=0 and y_{ij}=1. When y_{ij}=1, this problem can be rewritten as the following problem LG_{ij}^{21}.

$$(LG_{ij}^{21}) \qquad \pi_{ij}^{2} = \textit{minimize} \ \ a_{ij} f_{ij} + \sum_{d\in N}\sum_{o\in N}\left(v_i^{od} - v_j^{od}\right) x_{ij}^{od} \tag{27}$$
$$\textit{subject to} \quad (17) \textit{ and } (23)$$

Consequently the optimal value or the lower bound for *LG* is

$$\sum_{(i,j)\in A} min\{0, \pi_{ij}^{1}, \pi_{ij}^{2}\} + \sum_{d\in N}\sum_{o\in N}\left(v_d^{od} - v_o^{od}\right) - \sum_{d\in N}\sum_{i\in N\setminus\{d\}} w_i^{d} \ . \tag{28}$$

For $(i,j)\in A$, the optimal solution is y_{ij}=1 if $\pi_{ij}^{1}<0$ or $\pi_{ij}^{2}<0$ and y_{ij}=0 if not. For $o\in N$, $d\in N$, $(i,j)\in A$, the optimal value of x_{ij}^{od} is X_{ij}^{od1} if $\pi_{ij}^{1}\le\pi_{ij}^{2}$ and y_{ij}=1, X_{ij}^{od2} if $\pi_{ij}^{2}<\pi_{ij}^{1}$ and y_{ij}=1, and 0 otherwise, where X_{ij}^{od1} is the solution for LG_{ij}^{11} and X_{ij}^{od2} is the solution for LG_{ij}^{21}. Additionally, for $d\in N$, $(i,j)\in A$, the optimal solution is y_{ij}^{d}=1 if some $x_{ij}^{od}>0 (o\in N)$ and y_{ij}^{d}=0 if not, because ***w***=**0**. From these expressions, we can solve the Lagrangian relaxation problem *LG* and obtain the lower bound for the load planning problem *LTL*.

A MULTIPLIER ADJUSTMENT AND A SUBGRADIENT METHOD

We develop the multiplier adjustment method for setting the value of ***w***. Increasing the value of ***w*** from **0,** while ***w*** is feasible and the solution ***x*** and ***y*** for *LG* do not change, we could also increase the lower bound for *LTL*.

For LG_{ij}^{1}, let
$$\phi_i^{1} = min\Big\{ min\left(a_{ij} q^{od}/e_{ij} y_{ij} + v_i^{od} - v_j^{od} \mid x_{ij}^{od} = 0, o\in N, d\in N\right) - max\left(a_{ij} q^{od}/e_{ij} y_{ij} + v_i^{od} - v_j^{od} \mid x_{ij}^{od} > 0, o\in N, d\in N\right) \Big| \ j\in N \Big\} \quad i\in N\ . \tag{29}$$

For $LG_{ij}{}^2$, let $\phi_i^2 = min\{min(v_i^{od} - v_j^{od} \mid x_{ij}^{od} = 0, o \in N, d \in N)$

$$- max(v_i^{od} - v_j^{od} \mid x_{ij}^{od} > 0, o \in N, d \in N) \mid j \in N\} \quad i \in N . \tag{30}$$

Then we set the increment value of $\boldsymbol{w}$ as

$$w_i^d := max\{0, min(\phi_i^1, \phi_i^2, \varphi_i^{1d}, \varphi_i^{2d})\} \quad i \in N \setminus \{d\}, d \in N, \quad if \ K_i^d \geq 2 , \tag{31}$$

where

$$\varphi_i^{1d} = min\{-\sum_{d \in N} \sum_{o \in N} (a_{ij} q^{od} / e_{ij} y_{ij} + v_i^{od} - v_j^{od}) x_{ij}^{od} - \sum_{d' \in N \setminus [d]} w_i^{d'} \mid j \in N\} \tag{32}$$

$$\varphi_i^{2d} = min\{-a_{ij} f_{ij} - \sum_{d \in N} \sum_{o \in N} (v_i^{od} - v_j^{od}) x_{ij}^{od} - \sum_{d' \in N \setminus [d]} w_i^{d'} \mid j \in N\} \tag{33}$$

$$K_i^d = \{\text{the number of } x_{ij}^{od} \mid x_{ij}^{od} > 0, o \in N, j \in N\}. \tag{34}$$

When the values of $\boldsymbol{w}$ ascend up to these values, $\boldsymbol{w}$ is feasible because $w_i^d \geq 0$, and the optimal solutions for LG still do not change. Then the lower bound can be increased as much as

$$\sum_{d \in N} \sum_{i \in N \setminus \{d\}} w_i^d (K_i^d - 1). \tag{35}$$

The first term $w_i^d K_i^d$ is the increment value of the second term in (12) and the second term $-w_i^d$ is the decrement value of the fourth term in (12).

For setting the values of $\boldsymbol{v}$ approximately, we apply the standard subgradient optimization procedure (Fisher,1981). This is an iterative procedure, which uses the current multipliers $\boldsymbol{v}$**,** the current lower bound and an upper bound, in order to compute the new multipliers $\boldsymbol{v}$ used in the next iteration. Subgradient $\boldsymbol{g}$ of $\boldsymbol{v}$ can be defined follows,

$$g_n^{od} = \delta_n^{od} - \sum_{i \in N} x_{in}^{od} + \sum_{j \in N} x_{nj}^{od} \quad n \in N, o \in N, d \in N . \tag{36}$$

Then, using a step size s^t in iteration t, the new set of multipliers are given by

$$v_n^{od} := v_n^{od} + s^t g_n^{od} \quad n \in N, o \in N, d \in N . \tag{37}$$

It can be shown for a finite cardinality, if the step size s^t is selected so that $\lim_{t \to \infty} s^t = 0$,while $\sum_{t=0}^{\infty} s^t = \infty$, then the sequence $\boldsymbol{v}$ converges to the optimal value. We use the step size as

$$s^t = p^t \times (\text{the best known upperbound - the current lower bound}) / \|\mathbf{g}\|^2 \tag{38}$$

where p^t is a scalar which is initially equal to 1 and is reduced every some iteration number.

NUMERICAL EXPERIMENTS

In order to test the performance of our Lagrangian relaxation method, a set of numerical experiments is carried out using IBM compatible computer with PENTIUM4 1.7GHz, memory 256Mb and OS Windows 2000. This solution method is coded in COMPAQ VISUAL FORTRAN Ver.6. The problem data used in these experiments is randomly generated up to 50 nodes. N, the set of nodes presented end-of-line and break-bulk terminals is drawn from a uniform distribution over a rectangle measuring 100 by 100. A, the set of all potential links of direct services is $N \times N$. The line-haul cost per trailer on the link is in proportion to the Euclidean distance between nodes. Each of LTL freight demand is 1, the minimum frequency is 1 and the load capacity is $|N|$.

Table 1. Network Dimensions and Gaps

$\|N\|$	|A|	Commodity	Gap(%)
10	90	90	1.90%
20	380	380	3.39%
30	870	870	3.72%
40	1560	1560	5.31%
50	2450	2450	6.32%

Obtaining for an upper bound and approximate solutions, we use three kinds of Lagrangian heuristic algorithms (Katayama,2002), which

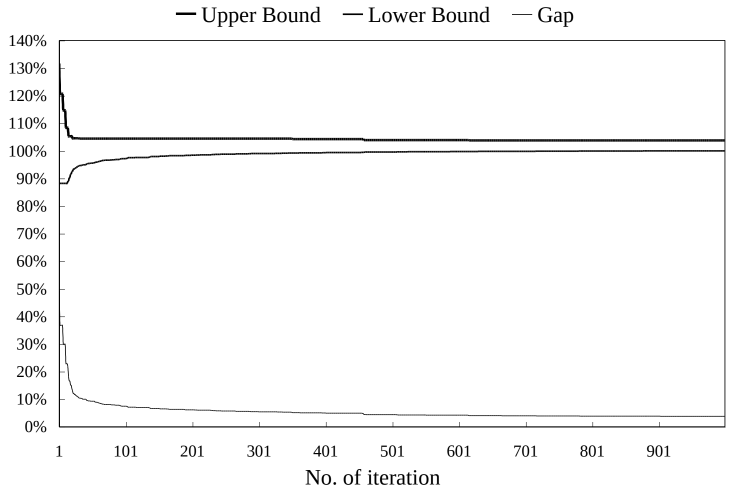

Figure 3. Convergence of method for 30 nodes

are a link delete heuristic, a successor matrix modification heuristic and a tabu search method.

Table 1 briefly summarizes the effectiveness of the Lagrangian relaxation method. It shows the number of nodes, potential direct services, commodity and the percentage gap between the best upper bound and the best lower bound. The percentage gaps range from 1.90% to 6.32%. Figure 3 shows the rate of convergence for the problem with 30 nodes. The subgradient optimization algorithm exhibited the fastest rate of convergence.

CONCLUSIONS

In this paper, we developed the load planning problem for LTL motor carriers and its solution method using the Lagrangian relaxation. The result of the experiments suggest that our Lagrangian relaxation problem and solution approach can perform a good job of identifying a lower bound of the load planning problem for LTL motor carriers. This research is underway to adapt solutions to the real world problems, such as the empty trailer balancing, the transit time and the number of transshipment, etc.

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