

A MODEL FOR THE OPTIMAL NUMBER AND LOCATIONS OF PUBLIC DISTRIBUTION CENTERS AND ITS APPLICATION TO THE TOKYO METROPOLITAN AREA

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In Japan, distribution centers managed by public authorities have been designed to achieve global optimization for efficient logistics and to maintain the Japanese urban environment. From the standpoint of social logistics, global optimization should be pursued considering both the environment and the efficiency of business logistics. In this study, we propose two nonlinear mathematical programming models for obtaining the optimal number and locations of public distribution centers and elaborate on the algorithm for determining solutions. These models aim at reducing the amount of truck CO₂ emissions while minimizing logistics costs, which consist of transportation, delivery and facility operating costs. Our models are applied to the Tokyo metropolitan area. As the result of that, an appropriate policy for the number and locations of public distribution centers is proposed.

Significance: The significance of this study is that 1) we deal with the location problem that focuses on the concave facility operating cost function and the environmental aspects, and develop an algorithm for solving this problem; 2) we aim at the global optimization of social logistics.

Keywords: Location, Public distribution center, CO₂ emissions, Mathematical programming, Approximate algorithm

1. INTRODUCTION

Distribution centers managed by public authorities in Japan have been designed to achieve global optimization for efficient logistics and to maintain the urban environment. This is an attempt to support global optimization of social logistics. From the standpoint of social logistics, global optimization should be pursued considering both the environment and the efficiency of business logistics. It is necessary to reduce the amount of CO₂ emitted by trucks, while minimizing logistics costs.

There are two purposes for the public distribution centers. Firstly, public distribution centers can be regarded as a solution for reducing logistics costs and realizing regional global optimization. Primarily, this could be achieved by reducing transportation and operating costs for distribution centers. Secondly, we expect public distribution centers to have a positive impact on the urban environment because their use results in a reduction of truck CO₂ emissions. The achievements are a result of fewer vehicles and shorter delivery distances. With these advantages, global optimization within an area could be realized.

In this study, we propose two mathematical programming models for obtaining the optimal number and locations of public distribution centers in the Tokyo metropolitan area. We first present existing studies dealing with location problems of distribution centers, and then we demonstrate the procedural description in Section 2. A location model and an algorithm for obtaining solutions to minimize total logistics costs within the area are presented in 3.1 and 3.2. In 3.3, a location model for minimizing the amount of CO₂ emissions is described, and solutions by a cost-minimizing problem are reconsidered in terms of the environment. These models are applied to the Tokyo metropolitan area, and an appropriate policy for public distribution centers is proposed in Section 4. Finally, the optimal number and locations of public distribution centers are obtained considering both logistics costs and the environment.

2. RELATED PROBLEMS AND PROCEDURAL DESCRIPTION

A wide variety of algorithms and applications for location problems have been proposed. Mirchandani and Francis (1990), and Daskin (1995) provide comprehensive reviews of discrete location problem methodologies including many exact and heuristic algorithms. The model, of which the objective function consists of fixed facility-operating costs and linear transportation costs, is called the simple facility location problem. For this problem, Bilde and Krarup (1977), and Erlenkotter (1978) proposed dual ascent algorithms and local search algorithms with these dual solutions. These algorithms can handle relatively large-scale problems efficiently. For the model with capacitated facilities, Pirkul (1987) proposed a Lagrangian relaxation algorithm. Many other facility location problems exist for minimizing costs. Furthermore, public service location problems have been proposed, and they are known as p -median problems. ReVelle and Swain (1970) proposed a relaxed linear programming algorithm, and Galvao (1989) proposed a Lagrangian relaxation algorithm for such a problem. Though almost all of location problems emphasize transportation costs, linear facility operating costs, or public services, there are few studies on the location problem that focus on environmental aspects. Taniguchi et al (1998) and Yamada et al (1999) presented location models with cost-minimizing, CO₂-minimizing and multiobjective functions. Although they considered traffic assignments, they applied it to a rather small-scale location model and did not deal with nonlinear operating costs. In our study, we deal with this type of location problems and elaborate on the algorithm for determining approximate solutions.

We conducted a survey (Industrial Research Institute, 1998) on the use of public distribution centers by polling 531 firms in the Tokyo metropolitan area. The survey indicates that approximately 60% of the participants wanted to make use of public distribution centers. To be precise, 70% of 167 wholesalers, 56% of 199 retailers and 54% of 165 manufacturers are expecting public distribution centers to be established. According to the survey, most of the firms were interested in the availability of land and use fees for distribution centers.

When considering the impact of public distribution centers on the environment, the sum of the vehicle-kilometers (the number of vehicles \times delivery distances) in the metropolitan area should be minimized to reduce truck CO₂ emissions. In order to do that, the number of distribution centers could be increased so as to decrease the total delivery distances. However, many companies tend to prefer to reduce the number of distribution centers to minimize stock volumes and costs.

We asked the survey participants about their views for the future regarding the number of distribution centers. Fifty-nine percent of the firms predicted a gradual integration while only nine percent predicted a more immediate increase in the number of distribution centers.

Furthermore, most companies actually do their best to reduce logistics costs rather than the sum of vehicle-kilometers or CO₂ emissions. Transportation costs from supply points to distribution centers, delivery costs from distribution centers to customers and operating costs in the centers are included in these logistics costs. As the total logistics costs within the area are summed up by each individual case, we first deal with the cost-minimizing problem and then consider the problem of reducing CO₂ emissions. Our study was carried out by following steps:

1. Build a mathematical model and an algorithm for considering the problem of minimizing logistics costs, namely, the sum of transportation, delivery and operating costs.
2. Evaluate solutions to this cost-minimizing problem by measuring the amount of CO₂ emissions.
3. Build a mathematical model and an algorithm for minimizing the amount of CO₂ emissions within the area.
4. Evaluate solutions to this CO₂-minimizing problem by measuring logistics costs.
5. Reconsider the number and locations of public distribution centers by both logistics costs and CO₂ emissions.

These procedures are conducted in and applied to the Tokyo metropolitan area.

3. MODEL AND ALGORITHM

3.1 A Location Model for the Logistics Cost-Minimizing Problem (CMP)

We first present a mathematical programming model for minimizing total logistics costs.

The following set of assumptions were made in order to formulate the model:

1. Supply points, demand points and candidate distribution center points are given as a set of nodes.
2. Distribution centers can handle unlimited amounts of any kind of commodities.
3. The amounts of supply and demand by goods at all nodes are given.
4. All goods from supply points are transported to demand points via a distribution center.
5. Transported volume from supply points to demand points is given.
6. Operating costs at distribution centers are increased nonlinearly by the amount of goods handled.

Although the assumption of handling unlimited amounts at distribution centers is not realistic, the size of public distribution centers is largely enough, and also able to be easily expanded in comparison with private distribution centers. Furthermore this assumption makes the model easy to solve.

We define the following notation to formulate our model. K is the commodity set, S is the supply point set, F is the candidate distribution center set and D is the demand point set. x_{ij}^k is a variable, which is the amount of transport of commodity k from supply node i to center j . z_j^k is a variable, which is the amount of delivery of commodity k from center j to

demand node l . Let y_j be a binary variable, which is equal to one if candidate center j is located or zero otherwise. u is the number of public distribution centers. d_{il}^k is the demand of commodity k transported from supply node i to demand node l ; c_{ij}^k is the transportation cost per volume, and e_{jl}^k is the delivery cost per volume. f_j is the nonlinear operating cost function at center j , and b_j^k is the amount of demand of commodity k handled at center j . M is the very large arbitrary number.

The problem is to determine the number and locations of public distribution centers to minimize total logistics costs. Using the above-mentioned notation, we can formulate an integer nonlinear mathematical programming model as follows:

$$\text{minimize } \sum_{i \in S} \sum_{j \in F} \sum_{k \in K} c_{ij}^k x_{ij}^k + \sum_{j \in F} \sum_{l \in D} \sum_{k \in K} e_{jl}^k z_{jl}^k + \sum_{j \in F} f_j \left(\sum_{k \in K} b_j^k \right) y_j \quad \dots (1)$$

$$\text{subject to } \sum_{j \in F} x_{ij}^k = \sum_{l \in D} d_{il}^k \quad i \in S, k \in K \quad \dots (2)$$

$$\sum_{j \in F} z_{jl}^k = \sum_{i \in S} d_{il}^k \quad l \in D, k \in K \quad \dots (3)$$

$$\sum_{i \in S} x_{ij}^k = \sum_{l \in D} z_{jl}^k = b_j^k \quad j \in F, k \in K \quad \dots (4)$$

$$x_{ij}^k \leq M y_j \quad i \in S, j \in F, k \in K \quad \dots (5)$$

$$z_{jl}^k \leq M y_j \quad j \in F, l \in D, k \in K \quad \dots (6)$$

$$\sum_{j \in F} y_j = u \quad \dots (7)$$

$$x_{ij}^k \geq 0 \quad i \in S, j \in F, k \in K \quad \dots (8)$$

$$z_{jl}^k \geq 0 \quad j \in F, l \in D, k \in K \quad \dots (9)$$

$$y_j \in \{0,1\} \quad j \in F \quad \dots (10)$$

$$u \in \text{integer} \quad \dots (11)$$

Equation (1) is the objective function, which should be minimized. The first term in this equation represents the transportation costs from supply points to the distribution centers. The second term is the delivery costs from distribution centers to demand points. The third term is the operating costs at these centers, which depends nonlinearly on the volume of handled goods. Equations (2), (3) and (4) are the conservation constraints; (2) is the one from supply points to distribution centers; (3) is the one from distribution centers to demand points, and (4) is the one at distribution centers respectively. Equations (5) and (6) are the forcing constraints, which show that any supply or demand must not be transported to center j or delivered from center j , if the center is not located in node j , that is, $y_j=0$. Equation (7) shows the relationship between y_j and u . The rest of the constraints, (8), (9), (10) and (11) denote nonnegative, zero-one or integer conditions.

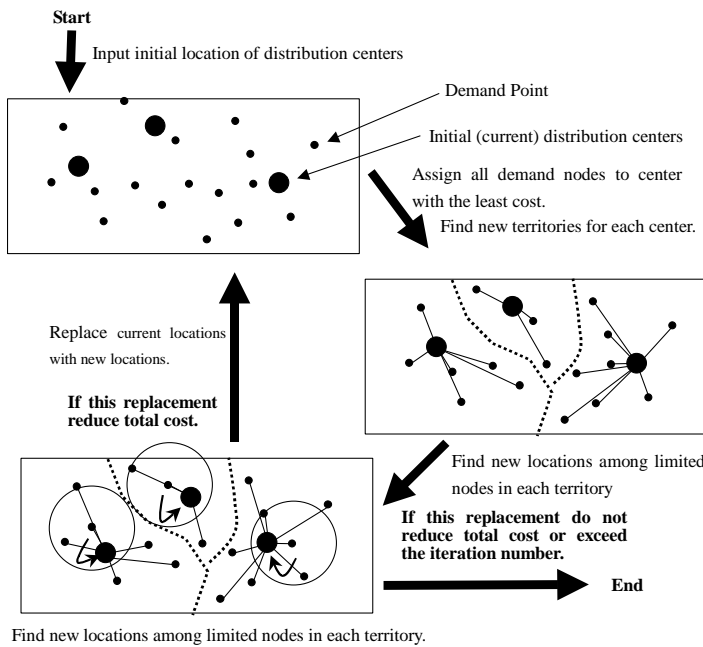


Figure 1. The limited neighborhood search improvement algorithm

3.2 Algorithm for CMP

Finding an optimal solution to this problem is very difficult because of its integrality and nonlinearity. Therefore, we adopted a heuristic algorithm to obtain a good approximate solution.

Given the number of public distribution centers and their temporal locations initially, each demand node is assigned to the center node with the lowest delivery cost, since we assume that distribution centers are uncapacitated and delivery costs should be minimized. A created set of nodes assigned to the same center forms a new territory for the center. We expect the new center serving each demand node in the territory to be located at the optimal node, minimizing the total cost in each territory. This problem is known as the one-median problem. We find these new locations by a simple enumeration method. Then, we replace current locations with new locations. While the total cost is reduced, the procedure is repeated to find new territories and new center locations, and to replace the old ones with new ones. This method is a local search or a neighborhood search improvement algorithm

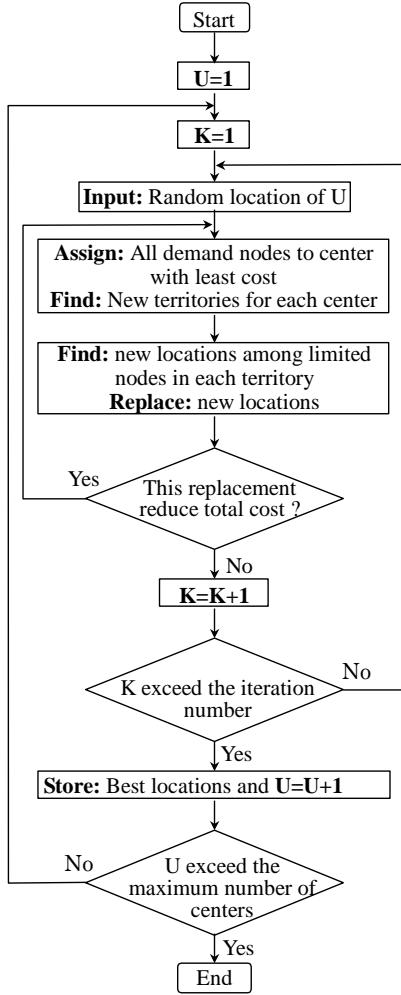


Figure 2. The RMLN algorithm for CMP

emitted through delivery activities from distribution centers to demand points. $g^k m_{ij}^k$ and $h^k n_{jl}^k$ denote coefficients to transform unit volume of transport or delivery into the volume of CO₂ emissions, respectively.

3.3 A Location Model for the CO₂-Minimizing Problem (CO₂MP)

As well as a location model for minimizing the logistics cost, we can easily formulate a model for the CO₂-minimizing problem using the proportional relationship between the vehicle-kilometers and the amount of truck CO₂ emissions. The objective function in CMP is the total cost of logistics and minimizing the logistics costs has been a traditional goal for most companies. However, the real goal for companies should be to improve the environment as well as to minimize costs. At this point, we propose another location model, which indicates how much the CO₂ emissions can be reduced.

Let the objective function of this problem be the amount of CO₂ emissions that should be minimized. When considering the amount of CO₂ emissions in the transportation and delivery activities, the objective function is the same as in the formula used for evaluating the amount of CO₂ emissions in the CMP. The constraints of CO₂MP are also the same as those of CMP.

We can formulate CO₂MP as the following integer mathematical programming model.

$$\begin{aligned}
 & \text{minimize} \sum_{i \in S} \sum_{j \in F} \sum_{k \in K} g^k m_{ij}^k x_{ij}^k + \sum_{j \in F} \sum_{l \in D} \sum_{k \in K} h^k n_{jl}^k z_{jl}^k \quad \dots (13) \\
 & \text{subject to} \quad (2), (3), \dots, (11)
 \end{aligned}$$

This problem could be regarded as a variation of the p -median problems. It is known that this type of problem is hard to solve exactly, as are real-world cases in large-scale applications. Therefore, we adopted a heuristic algorithm to obtain a good approximate solution. As this problem is the same as CMP, except for the objective function in which we substitute the amount of CO₂ emissions for the total logistics costs in the one-median problem for each territory, we can apply our RMLN algorithm.

(Daskin, 1995).

In this neighborhood search, as an enumeration of every node within each territory brings about a considerable change in territories and center locations, the approximate total cost in the one-median problem might often be inaccurate. Therefore, we limited the enumeration to a certain number of nodes with lower delivery costs rather than including all nodes in each one-median problem. Given the number and their initial locations of public distribution centers, Figure 1 illustrates this limited neighborhood search improvement algorithm.

Since the solution obtained by the neighborhood search is strongly dependent on its initial solution, we should provide a large number of random initial locations and repeat the neighborhood search according to a variety of initial location, which is called multi-start algorithm (Brimberg, 1997). Furthermore, as the number of public distribution centers could vary within the appropriate range up to the upper limit, we repeat the neighborhood search for every number of centers. In this way, we can obtain the best number of public distribution centers and locations from among all approximate solutions. Figure 2 shows a flowchart of this random multi-start limited neighborhood search (RMLN) algorithm. In Figure 2, U denotes the current number of centers and K denotes the current iteration number at U centers.

Solutions obtained by the above-mentioned algorithm are evaluated in terms of the amount of CO₂ emissions to take the environmental aspects into consideration. As the amount of CO₂ discharged by trucks is considered in proportion to the sum of the vehicle-kilometers within the area, we substitute these solutions for the following formula and evaluate the amount of CO₂ emissions in case of these solutions.

$$\sum_{i \in S} \sum_{j \in F} \sum_{k \in K} g^k m_{ij}^k x_{ij}^k + \sum_{j \in F} \sum_{l \in D} \sum_{k \in K} h^k n_{jl}^k z_{jl}^k \quad \dots (12)$$

where, g^k is the average volume of CO₂ emissions per vehicle for transport of commodity k , h^k is the average volume of CO₂ emissions per vehicle for delivery of commodity k . m_{ij}^k is the number of vehicles per volume transported from supply node i to distribution center j of commodity k , and n_{jl}^k is the number of vehicles per volume delivered from distribution center j to demand node l of commodity k .

This formula represents the amount of CO₂ emissions within the area. The first term in formula (12) is the amount of CO₂ emitted through transportation activities from supply points to distribution centers. The second term is the amount of CO₂

4. APPLICATIONS

We apply our models to the Tokyo metropolitan area. The Tokyo metropolitan area consists of the Tokyo metropolis and the Kanagawa, Chiba and Saitama prefectures. The Tokyo metropolitan area is the center of the Japanese economy, transportation and delivery activities, and more than half of CO₂ emissions in Japan are discharged in this area.

4.1 Data and Assumptions

Our applications are based on the following conditions. We divided the area into 283 demand points, which are located at the centers of the cities, towns and villages comprising the Tokyo metropolitan area. It is assumed that the distribution centers are to be located somewhere among these demand nodes. The number of supply points is nine, and they are located at the entrance nodes outside the Tokyo metropolitan area and in nodes with main sources within that area. We deal with the following eight kinds of commodities: agricultural and marine, forest, mineral, metal and machine, chemical, light industrial, other industrial and special. Demand data, from every supply point to every demand point by commodity, are obtained from a Goods Flow Survey of the Tokyo metropolitan area (1997).

The transformation coefficient from vehicle-kilometer into the amount of CO₂ emissions is calculated from the CO₂ discharge coefficient of a gasoline vehicle and a diesel vehicle, and the ratio of a gasoline and a diesel vehicle in transportation and delivery activities. The number of vehicles per volume transported from the supply point to the distribution center and delivered from the distribution center to the demand point are calculated using average truck loading rates, loading capacities and volume of transport or delivery.

In the RMLN algorithm, the number of the public distribution center varies from one to 20, and the limited number of enumerating nodes at neighborhood search is ten. The iteration number of random initial locations is 2,500,000 times for every number of the public distribution center, and we repeat the neighborhood search by using these initial locations in the RMLN algorithm. We leave some other detailed assumptions and data to the appendix A.

4.2 Results of CMP and CO₂MP

Using the above-mentioned data and assumptions, we solved CMP by the RMLN algorithm and obtained the optimal number of public distribution centers and their locations in terms of logistics costs. Figure 3 shows the transportation and delivery, operating and total costs as the number of centers is increased. The total costs decrease until there are three centers and gradually increase from there on. The optimal number of the centers is three, as shown in Figure 3. We leave the convergence of RMLN algorithm and other intermediate solutions to the appendix B.

Since CMP is calculated on the basis of ton-kilometers, the amount of ton-kilometers is converted into the amount of CO₂ emissions by Equation (12). Figure 4 shows the amount of CO₂ discharged by trucks within the Tokyo metropolitan area as the number of centers is increased. In this case, it is clear that the amount of CO₂ emissions decreases sharply until there are four centers and the amounts seem to remain stable when there are more than four centers.

The number of public distribution centers should be determined based upon both total logistics costs and the amount of CO₂ emissions. The difference in the total costs between three and four centers is very small (0.3%), as shown in Figure 3. The difference in the amount of CO₂ emissions between three and four centers is 11.3%, as shown in Figure 4. Since the total costs are almost the same in these two cases, and the amount of CO₂ emissions can be reduced by about 11%, therefore, we

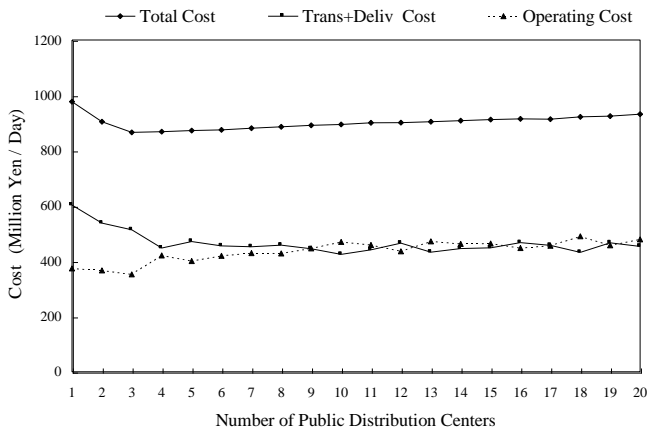


Figure 3. Costs versus number of public distribution centers

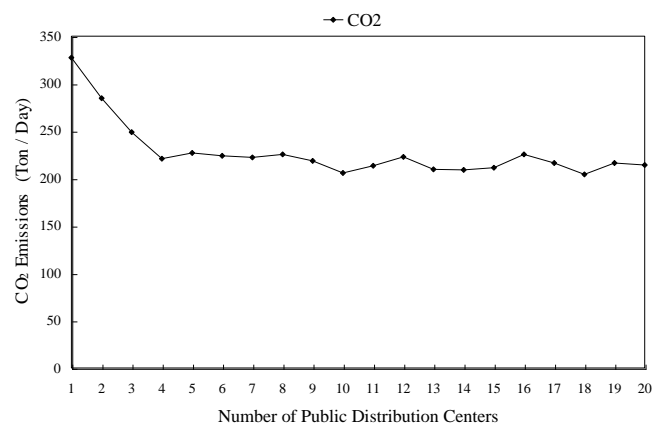


Figure 4. CO₂ Emissions versus number of public distribution centers

recommend four locations from the environmental standpoint. Figure 5 illustrates locations of four centers and their territories.

By the RMLN algorithm as described in 3.2, we solved CO₂MP, which is minimized by the sum of truck CO₂ emissions instead of by the total logistics costs. Figure 6 shows that the comparison of the total logistics costs between CMP and CO₂MP. The minimum cost in CO₂MP is obtained when there are four centers. As a matter of course, the total logistics costs in CO₂MP are higher than that in CMP. In case of three centers, the difference in the total costs accounts for 19.3%, 13.3% in case of four centers, and 17.1% in case of 20 centers. On the contrary, the amounts of CO₂ emissions in CO₂MP are lower than that in CMP. In both problems, the amounts of CO₂ emissions decrease sharply until there are four centers. After that, the amounts of the CO₂ emissions gradually decrease as shown in Figure 7. The difference in the amount of CO₂ emissions of both problems accounts for 15% in case of three centers, 13.9% in case of four centers and 41.1% in case of 20 centers. These results suggest that if all companies do their best to reduce CO₂ emissions rather than their logistics costs, the amount of CO₂ emissions could be reduced by about 14 to 40%.

In addition to CO₂ emitted proportional to the vehicle-kilometers, CO₂ is also discharged near the distribution centers by trucks idling. If the number of centers were too small, the amount of CO₂ emissions would increase because the size of the centers would increase, as would the likelihood of traffic jams. Our model does not consider this aspect of CO₂ emissions. Therefore, when we consider the environmental aspects, the number of centers should be further increased. In our applications, having four to nine centers seems to be the best option as the logistics costs do not increase as much, as shown in Figure 6.

5. CONCLUSION

In this paper, we formulated two models for the optimal location of public distribution centers and developed the RMLN algorithm for solving the nonlinear mathematical programming models. We applied our models to the Tokyo metropolitan area and presented the optimal policy for the number and locations of public distribution centers.

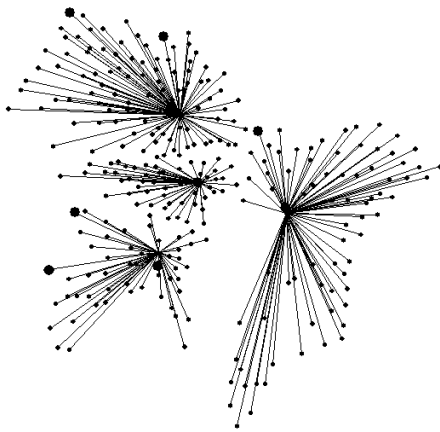


Figure 5. Locations of 4 centers and their territories

Although it is difficult to reduce the amount of CO₂ emissions, environmental issues should be considered as well as the logistics costs involved in the transportation and delivery of goods to achieve successful social or “green” logistics. Public distribution centers will attempt to achieve global optimization for the community.

In this area, there are 7 public distribution centers and 6 centers are currently under construction. However, these centers have been planned and constructed based mainly on the economic points of view. The application of our model has enabled us to suggest an appropriate policy for public distribution centers in the Tokyo metropolitan area. Furthermore, for the smooth integration of public distribution centers, we might examine methods employed by the PFI (Private Finance Initiative). Good examples of PFI for public utilities can be found in the United Kingdom. The Japanese Ministry of Economy, Trade and Industry is now examining the introduction of PFI for public distribution centers as a way of realizing the global optimization of logistics.

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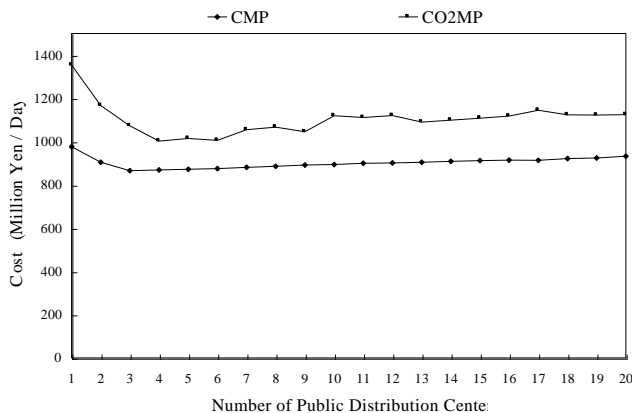


Figure 6. Total Costs for CMP and CO₂MP

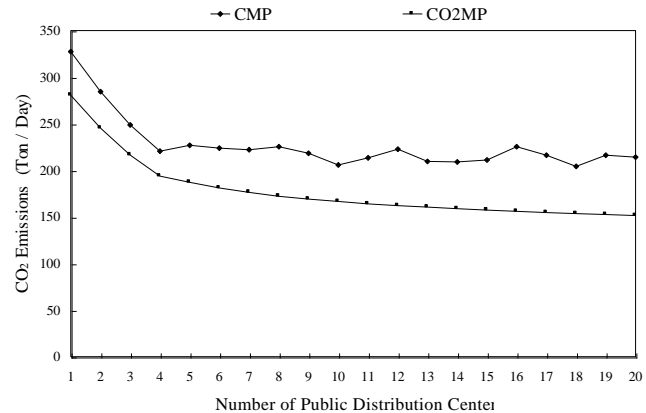


Figure 7. CO₂ Emissions Cost for CMP and CO₂MP

APPENDIX A : DETAILED ASSUMPTIONS AND DATA

The following assumptions for our applications are made concerning operating, transportation, delivery and land costs:

1. Operating cost function at a public distribution center is defined as:

$$\begin{aligned} & (\text{Land price per unit area at that site} \times \text{Required area per unit handled volume} \\ & + \text{Floor price per unit area} \times \text{Required floor space per unit handled volume}) \times \text{Handled volume} \\ & \times (\text{Total demand in the Tokyo metropolitan area} / \text{Handled volume at the center})^{0.25}. \end{aligned}$$

The last term of this formula represents the concavity of operating costs that are increased concavely as the number of centers is increased. This type of formula is known as a good approximate function in the real world (Kuse 1997).

2. Land price per area of a site is defined as the following function (Kuse 1997):

$$\max(a, b - c \times \log_{10}(\text{Euclidean distance to the center of Tokyo})).$$

Let a be the lowest bound of land price per area and b and c be constants for the approximation curve of land price.

3. Transportation costs is defined as: Distance from the supply point to the distribution center
 \times Volume of transportation in tons \times Unit transportation cost per ton \cdot kilometer by commodity.
4. Delivery costs is also defined as: Distance from the distribution center to the demand point
 \times Volume of delivery in tons \times Unit delivery cost per ton \cdot kilometer by commodity

In order to calculate these costs, we use the following input data in Table 1. Figure 8 illustrates locations of supply points and demand points in the Tokyo metropolitan area.

APPENDIX B : THE CONVERGENCE OF THE RLMN ALGORITHM AND INTERMEDIATE SOLUTIONS

Table 2 shows the improvement process in the total cost in case of four centers by using the RLMN algorithm for CMP. In the RMLN algorithm, the iteration number of random initial locations is 2,500,000 times for every number of the public distribution center. As Table 2 shows, it is clear that most of the improvement by this algorithm can be seen in the first 40 iterations. Figure 9 illustrates the improvement process of solutions: (a) locations and their territories of four public distribution centers at the first iteration, (b) at the 8th iteration and (c) at the 40th iteration.

Figure 10 illustrates locations and their territories of two, six and eight public distribution centers for CMP: (a) two centers, (b) four centers and (c) six centers. Figure 10 shows ratios of the total cost in each number of centers to that in case of four centers. Ratios of the total CO₂ emissions in each number of centers to that in case of four centers are shown in parentheses.

Table 1. Input Data

Required area of site per unit volume ($m^2/\text{ton} \cdot \text{day}$)	135	Land price function ($\text{yen}/m^2 \cdot \text{day}$)	$a=3.03$ $b=154.5$ $c=39.9$
Required floor space per unit volume ($m^2/\text{ton} \cdot \text{day}$)	107	Floor price ($\text{yen}/m^2 \cdot \text{day}$)	39.4
Average loading rate (transportation)	80.7%	The number of Demand data between supply points with demand points	20376
Average loading rate (delivery)	76.4%	Volume of CO ₂ emissions ($g/km \cdot \text{vehicle}$)	356
Average loading capacity (transportation) (ton)	7.16	Volume of CO ₂ emissions ($g/km \cdot \text{vehicle}$)	282
Average loading capacity (delivery) (ton)	4.59	Total demand (ton/day)	49867

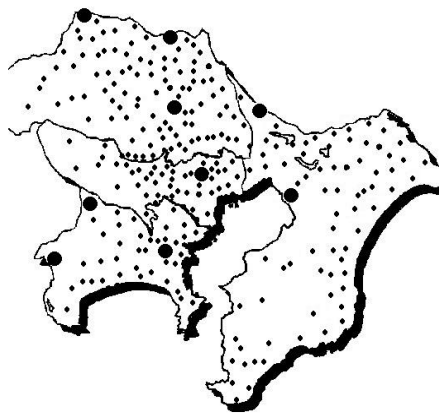


Figure 8. Locations of demand and supply points

Table 2. Convergence of the RMLN algorithm for CMP of 4 Centers

No. of Iterations	Improvement (%)	No. of Iterations	Improvement (%)	No. of Iterations	Improvement (%)	No. of Iterations	Improvement (%)
1	128.56	8	106.08	3348	101.13	50034	100.56
2	121.53	33	105.41	6019	100.97	347271	100.45
3	117.27	40	102.10	12396	100.95	365258	100.42
4	110.81	811	102.07	16345	100.87	507773	100.21
6	107.41	2552	101.72	38461	100.72	1108748	100.00

Improvement: $100 \times \text{current least cost} / \text{cost of the best solution}$; No. of iterations; K in Figure 2.

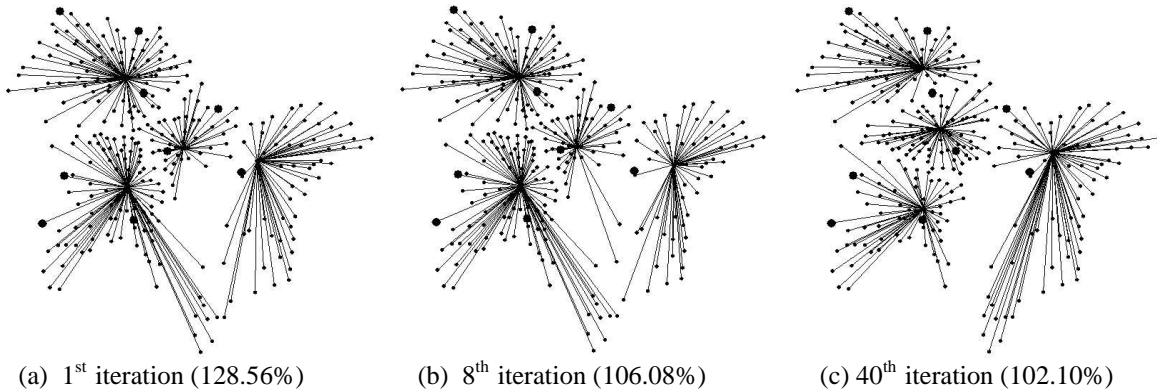


Figure 9. Locations of 4 Public Distribution Centers

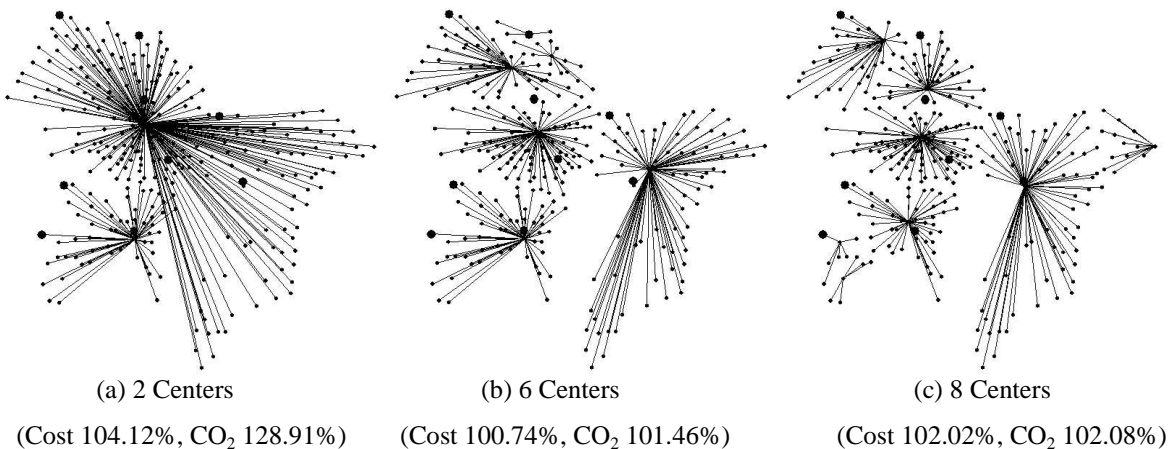


Figure 10. Locations of 2,6 and 8 Public Distribution Centers for CMP

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