A Capacity Scaling Procedure for the Multi-Commodity Capacitated Network Design Problem

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Introduction

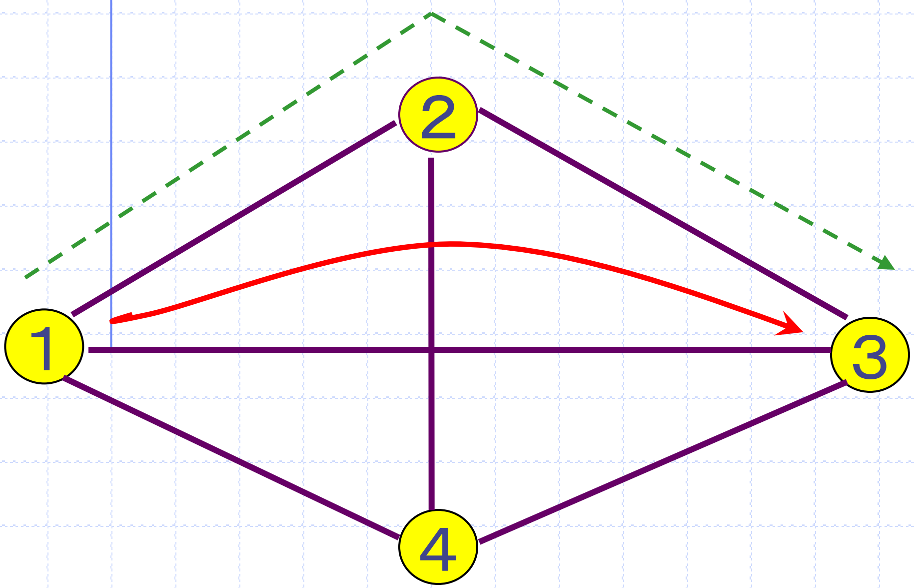
The multi-commodity capacitated network design problem:

- Demands of multi-commodity are given
- Arc capacities are limited
- The total of the design and flow costs is minimized
- The network is configured
- Flow paths are decided

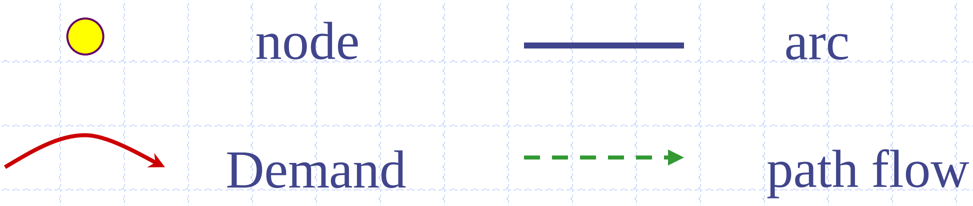
- Strong and weak, path type formulations
- Column generation technique
- Capacity scaling procedure

are proposed in this study

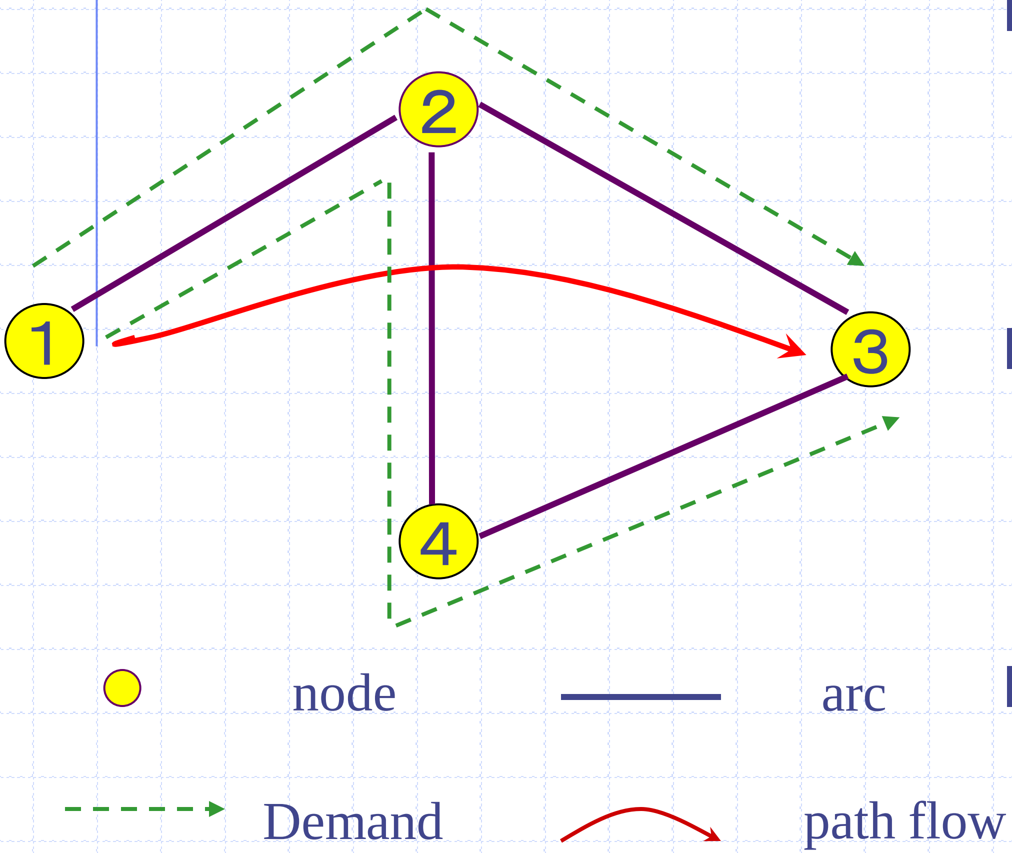
The multi-commodity capacitated network design model



- Node : Distribution center, break bulk point, end-of-line point, computer
- Arc : less-than-truckload service, feeder service, communication line
- Flow : Freights, communication data



The multi-commodity capacitated network design model



- Node : Distribution center, break bulk point, end-of-line point, computer
- Arc : less-than-truckload service, feeder service, communication line
- Flow : Freights, communication data

Literatures

- Crainic–Gendron–Hernu（2003）
  The slope scaling method, changing the flow cost on the weak formulation
  The slope scaling solution method by the Lagrange relaxation method and the long term memory
- Crainic–Gendreau–Farvolden（2000）
  The column generation technique
  The taboo search method for the base transformation at a simplex method
- Ghamlouche–Crainic–Gendreau（2002，2003)
  The cycle based neighborhood structure
  The path relinking method

Notations

- N :set of nodes
- A :set of arcs
- K :set of commodities
- P^k :set of paths from origins to destinations for commodity k
- P'^k:subset of path P^k
- d^k:demand for commodity k
- c_{ij}^k: unit cost for commodity on arc(i, j)

Notations

- f_{ij} :design cost of arc(i, j)
- C_{ij} :capacity of arc(i, j)
- δ_{ijp}:1, if arc (i, j) is included in path, otherwise 0.
- x_p^k :flow of commodity k on path p,
  continuous nonnegative variable
- y_{ij}:1, if arc (i, j) is selected, otherwise 0, binary variable

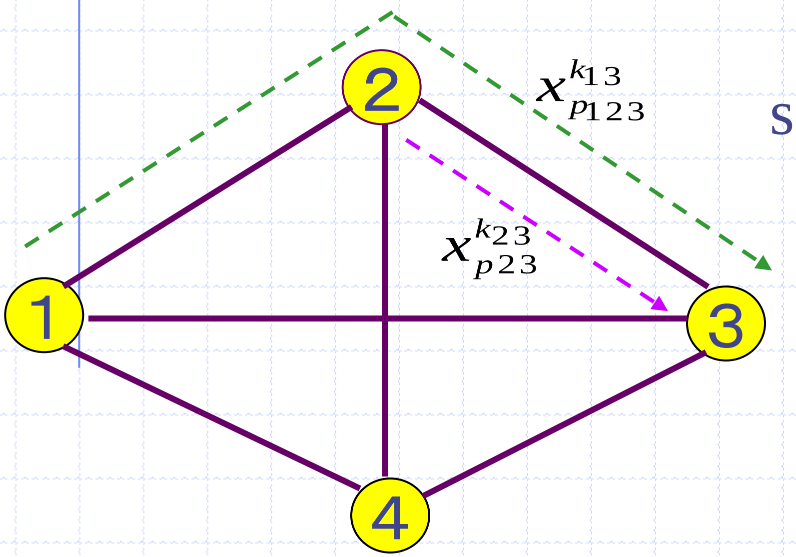
Strong Path flow Formulation

$$(MCND(P)) \quad \min \sum_{(i,j)\in A} \sum_{k\in A} c_{ij}^{k} \sum_{p\in P^{k}} \delta_{ijp} x_{p}^{k} + \sum_{(i,j)\in A} f_{ij} y_{ij}$$

Flow costs

Design costs of arcs

Formulation

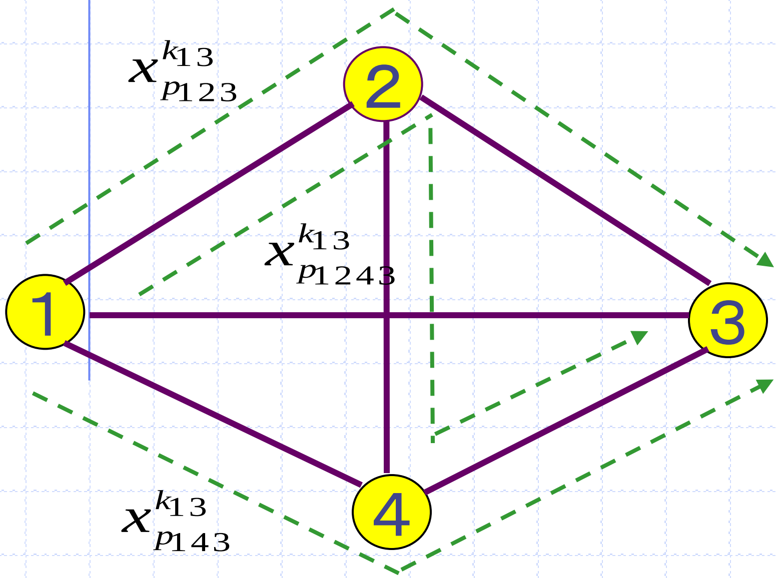


s.t. $$\sum_{k \in K} \sum_{p \in P^k} \delta_{ijp} x_p^k \le C_{ij} y_{ij} \quad (i, j) \in A$$

Flow on arc (i, j) for every commodity is limited by the arc capacity

$$x_{p123}^{k13} + x_{p23}^{k23} \le c_{23}$$

Formulation



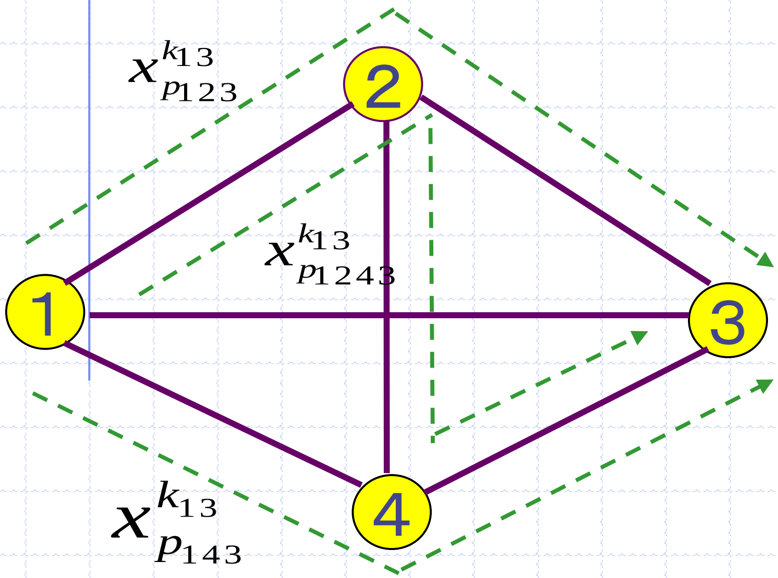
$$\sum_{p \in P^k} \delta_{ijp} x_p^k \le d^k y_{ij} \quad (i,j) \in A \;\; k \in K$$

Path flow on arc (*i*,*j*) for commodity *k* can not exceed its demand. Without these constraints MCND(P) becomes a weak formulation

$$x_{p123}^{k13} + x_{p1243}^{k13} \le d^{k13}$$

$$x_{p143}^{k13} \le d^{k13}$$

Formulation



$$\sum_{p \in P^k} x_p^k = d^k \qquad k \in K$$

The total of path flows for commodity *k* equals to its demand

$$x_{p123}^{k13} + x_{p1243}^{k13} + x_{p143}^{k13} = d^{k13}$$

Formulation

$$x_p^k \geq 0 \qquad k \in K \qquad p \in P^k$$

continuous nonnegative variables

$$y_{ij} \in \{0,1\} \qquad (i,j) \in A$$

The design variables are binary variables

Formulation using *P'*, subset of *P*

(*MCND*(*P'*)) min $\sum_{(i,j)\in A}\sum_{k\in A} c_{ij}^{k} \sum_{p\in P'^{k}} \delta_{ijp} x_{p}^{k} + \sum_{(i,j)\in A} f_{ij} y_{ij}$ (1)

s.t. $\sum_{k\in K}\sum_{p\in P'^{k}} \delta_{ijp} x_{p}^{k} \leq C_{ij} y_{ij} \qquad (i,j)\in A$ (2)

$\sum_{p\in P'^{k}} \delta_{ijp} x_{p}^{k} \leq d^{k} y_{ij} \qquad (i,j)\in A \qquad k\in K$ (3)

$\sum_{p\in P'^{k}} x_{p}^{k} = d^{k} \qquad k\in K$ (4)

$x_{p}^{k} \geq 0 \qquad k\in K \qquad p\in P'^{k}$ (5)

$y_{ij} \in \{0,1\} \qquad (i,j)\in A$ (6)

Without these constraints MCND(P) becomes a weak formulation

Capacity scaling procedure

- Solve the linear relaxation problem
  - Change the capacities of arcs based on solutions of LP relaxation and previous capacities and solve LP
  - Iterate above procedures, until the design variables *y* converge at 0 or 1
- Solve the multi-commodity flow problem with fixed design variable y, based on convergent solutions

$$\sum_{k \in K} \sum_{p \in P'^k} \delta_{ijp} x_p^k = x_{ij}^k$$

Capacity scaling procedure

- Relationship between *x* and *y*

$$\sum_{k \in K} \sum_{p \in P'^k} \delta_{ijp} x_p^k \le C_{ij} y_{ij} \quad x_p^k \ge 0 \quad y_{ij} \in \{0,1\} \rightarrow \text{Nonlinear}$$

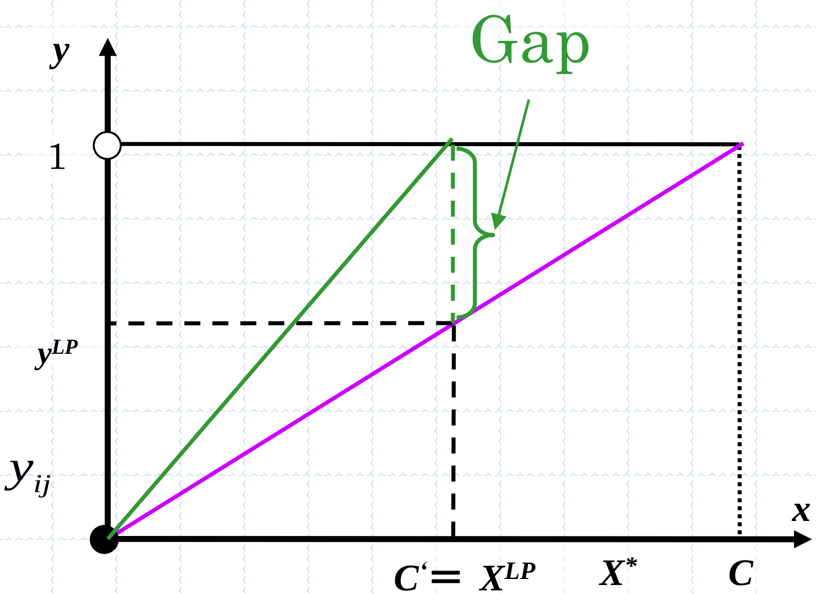
- Relax variable *y*

$$y_{ij} \in \{0,1\} \rightarrow 0 \le y_{ij} \le 1 \rightarrow \text{Linear}$$

- Change capacity *C’*

$$\sum_{k \in K} \sum_{p \in P'^k} \delta_{ijp} x_p^k \le C_{ij} y_{ij} \rightarrow C_{ij} y_{ij} \doteqdot x_{ij}^{LP}$$

$$y_{ij} = 1 \rightarrow \quad C'_{ij} = x_{ij}^{LP} \quad \rightarrow \sum_{k \in K} \sum_{p \in P'^k} \delta_{ijp} x_p^k \le C'_{ij} y_{ij}$$



- Once *C’* is less than the optimal solution *x**, the optimal solution can never be obtained.

Capacity scaling procedure

$$\sum_{k\in K}\sum_{p\in P^k}\delta_{ijp}x_p^k \le C'_{ij}\,y_{ij}$$

- Change capacity *C'* by the smoothing parameter λ

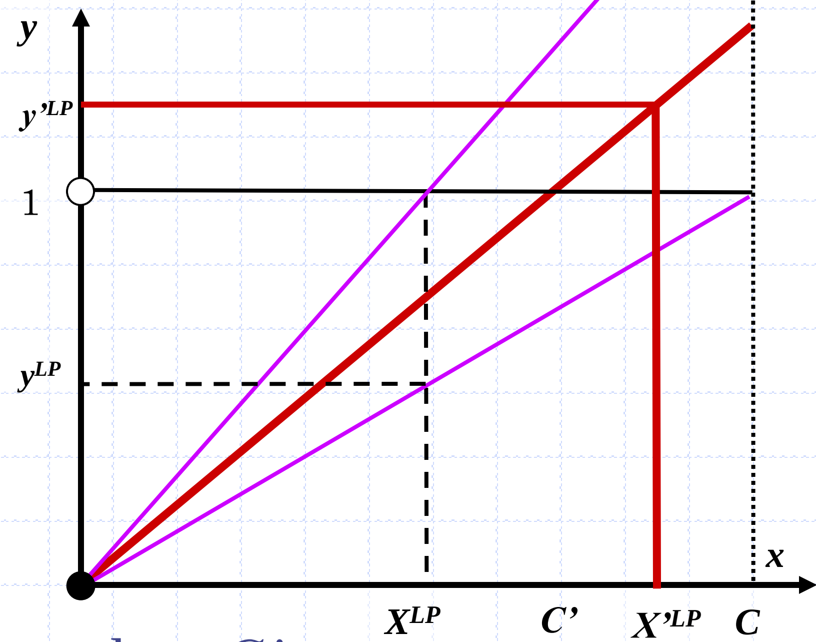
$$C'_{ij} = \lambda x_{ij}^{LP} + (1-\lambda)C'_{ij}$$

- Sudden changes of *C'* can be prevented

- Change the upper bound of *y*

$$0 \le x_{ij} \le C_{ij} \;\fallingdotseq\; 0 \le C'_{ij}\,y_{ij} \le C_{ij}$$

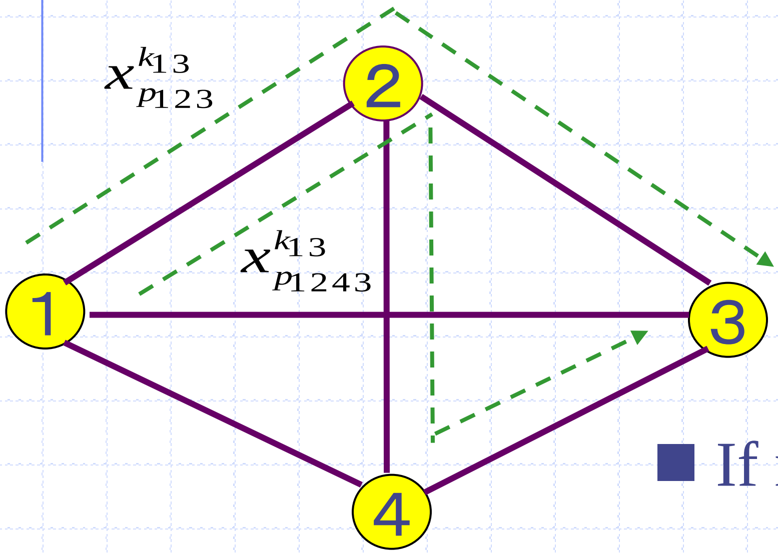
$$0 \le y_{ij} \le 1 \;\Rightarrow\; 0 \le y_{ij} \le \frac{C_{ij}}{C'_{ij}}$$



- x^{LP} can get the values which is greater than *C'*
- Prevent the situation that the real capacity is exceeded

Column generation

When the numbers of arcs and commodities become large, the number of paths may increase extremely



- At first, solve LP using the subset of paths, *P'*
- If necessary, generate paths (columns),
- Add columns to LP and solve LP again

Restricted primal problem

LP relaxation of (*MCND*(*P'*))

(*MCNDR*(*P'*)) min $\sum_{(i,j)\in A}\sum_{k\in A} c_{ij}^k \sum_{p\in P'^k} \delta_{ijp} x_p^k + \sum_{(i,j)\in A} f_{ij} y_{ij}$ (1)

Dual variables $\pi_{ij} \geqq 0$

s.t. $\sum_{k\in K}\sum_{p\in P'^k} \delta_{ijp} x_p^k \le C'_{ij}\, y_{ij} \quad (i,j)\in A$ (2)

Dual variables $\sigma^k_{ij} \geqq 0$

$\sum_{p\in P'^k} \delta_{ijp} x_p^k \le d^k y_{ij} \quad (i,j)\in A \quad k\in K$ (3)

Dual variables β^k

$\sum_{p\in P'^k} x_p^k = d^k \quad k\in K$ (4)

Relaxation and change of upper bounds

$x_p^k \ge 0 \quad k\in K \quad p\in P'^k$ (5)

$0 \le y_{ij} \le C_{ij} / C'_{ij} \quad (i,j)\in A$ (7)

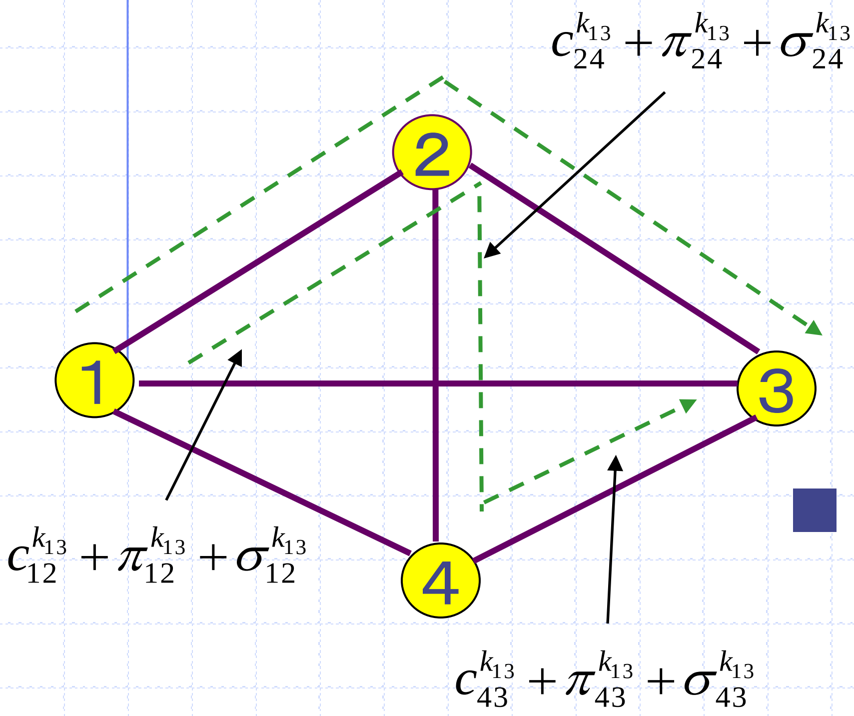
Column generation -Reduced cost

Using dual variables, substitute constraints (2),(3),(4) into the objective function

$$\sum_{k\in A}\sum_{p\in P'^k}[\sum_{(i,j)\in A}(c_{ij}^k+\pi_{ij}+\sigma_{ij}^k)\delta_{ijp}-\beta^k]x_p^k+\sum_{(i,j)\in A}(f_{ij}-\pi_{ij}C_{ij}-\sum_{k\in K}\sigma_{ij}^k d^k)y_{ij}+\sum_{k\in K}\beta^k d^k$$

- All coefficients of x : nonnegative at the optimal solution
- Coefficient : the reduced cost for x.
- Solve the pricing problem
- Decide whether the reduced cost is negative or nonnegative.

Column generation -Reduced cost



- Reduced cost for flow variable x^k_p

$$h_p^k = \sum_{(i,j)\in A} (c_{ij}^k + \pi_{ij} + \sigma_{ij}^k)\delta_{ijp} - \beta^k$$

Column generation - Pricing problem

For each commodity,

- Pricing problem : shortest path problem

$$\min \quad \sum_{(i,j)\in A} (c_{ij}^k + \pi_{ij} + \sigma_{ij}^k)\delta_{ijp}$$

$$\sum_{j\in N} \delta_{ijp} - \sum_{j\in N} \delta_{jip} = \begin{cases} -1 & i = O^k \\ 0 & i \in N \setminus \{O^k, D^k\} \\ 1 & i = D^k \end{cases}$$

$$\delta_{ijp} \in \{0,1\} \qquad (i,j)\in A \qquad p \in P'^k$$

- If the reduced cost is negative, then this shortest path is added as a path flow variable.

Procedure of column generation

1) Obtain the set of paths for every commodity k, and defined it as P'^k
2) Solve $MCNDR(P')$, obtain dual variables
   π_{ij} ,σ^k_{ij} ,β_k
3) For all commodities do
   a) Let the arc cost be $(c^k_{ij}+\pi_{ij} + \sigma^k_{ij})$, and solve the shortest path problem for commodity k, then obtain the value of h^k_p
   b) If $h^k_p < 0$, add the path to P'^k
4) If paths are add to P, go to 2); Otherwise, end

Row generation

- The number of forcing constraints is depended on the product space of arcs and commodities

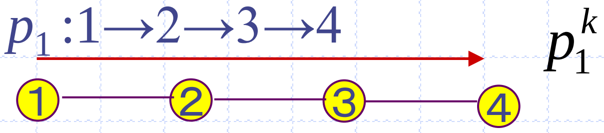
$$\sum_{p \in P'^k} \delta_{ijp} x_p^k \le d^k y_{ij} \qquad (i,j) \in A \qquad k \in K \qquad (3)$$

- For example, 500 arcs, 100 commodities, forcing constraints is up to 50000
- When columns are generated, forcing constraints (rows) correspond to columns are added.

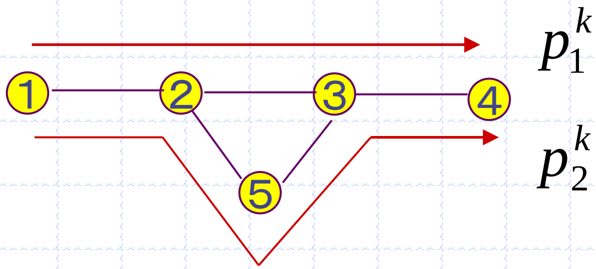
Row generation

Initial path for commodity k

$p_1: 1 \rightarrow 2 \rightarrow 3 \rightarrow 4$



Generated path $p_2: 1 \rightarrow 2 \rightarrow 5 \rightarrow 3 \rightarrow 4$



$$\begin{cases} x^k_{p_1} \le d^k y_{12} \\ x^k_{p_1} \le d^k y_{23} \\ x^k_{p_1} \le d^k y_{34} \end{cases}$$

$$\sum_{p \in P'^k} \delta_{ijp} x^k_p \le d^k y_{ij}$$

$$x^k_{p_1} + x^k_{p_2} \le d^k y_{12}$$

$$x^k_{p_1} \le d^k y_{23}$$

$$x^k_{p_1} + x^k_{p_2} \le d^k y_{34}$$

$$x^k_{p_2} \le d^k y_{25}$$

$$x^k_{p_2} \le d^k y_{53}$$

Generated column

Generated rows

Capacity scaling procedure

Let MCNDR(C) be the problem with arc capacity C

1) Define the smoothing parameter as $\lambda \in (0,1]$, and let C' be C

2) Iterate to solve MCNDR(C'), until y converges at 0 or 1

   a) Change the domain of y to $[0, \; C/C']$

   b) Solve MCNDR(C') by column generation, obtain solutions x, y

   c) Update $C' := \lambda C' y + (1 - \lambda) C'$

3) Obtain the approximate solution and the upper bound from the convergence solution.

Numerical Experiments

Experiment environments

- Data: Multi-commodity Network Design Problems in OR-Library: Problem Mulgen I from p33 to p50
- Computer：Pentium 4, 2.8GHz, RAM 750Mb
- Language: C
- Software for solving LP：GLPK ver.4.1
- Lower bounds by CPLEX (max CPU time 10 hours) for gaps

Computational Results

Table 1. Computational results

Names of Problems	Number of Nodes	Number of Arcs	Number of Commodities	The weak		The strong	
				gaps	time	gaps	time
p33	20	230	40	0.3%	0:00:02	0.0%	0:00:03
p34	20	230	40	3.3%	0:00:03	0.0%	0:00:04
p35	20	230	40	0.2%	0:00:03	0.0%	0:00:03
p36	20	230	40	1.9%	0:00:03	0.6%	0:00:04
p37	20	229	200	9.1%	0:00:08	4.8%	1:27:42
p38	20	229	200	7.6%	0:00:07	3.4%	8:36:00
p39	20	229	200	10.5%	0:00:07	2.6%	1:27:03
p40	20	229	200	10.8%	0:00:11	5.1%	14:42:02
p41	20	289	40	0.8%	0:00:04	0.3%	0:00:05
p42	20	289	40	2.4%	0:00:04	0.1%	0:00:08
p43	20	289	40	0.6%	0:00:02	0.3%	0:00:03
p44	20	289	40	2.3%	0:00:03	0.6%	0:00:04
p45	20	287	200	11.4%	0:00:08	3.9%	1:48:42
p46	20	287	200	8.3%	0:00:12	6.0%	11:54:52
p47	20	287	200	9.6%	0:00:07	3.3%	0:57:36
p48	20	287	200	9.2%	0:00:14	6.8%	7:43:39
p49	30	517	100	3.6%	0:00:09	1.1%	0:01:42
p50	30	517	100	18.4%	0:00:04	6.9%	51:24:35

Computational time: hours:minutes:seconds

Conclusion

This study considered the multi-commodity capacitated network design problem

- Path type weak and strong formulations were presented
- A capacity scaling procedure combined with column generation was proposed
- The effectiveness of the formulation and solution procedure was tested by numerical experiments