A SUPPLY CHAIN NETWORK DESIGN PROBLEM
WITH CONCAVE INVENTORY COSTS

S. Tei, N. Katayama, S. Yurimoto

Distribution and Logistics Systems, Ryutsu Keizai University 120 Hirahata, Ryugasaki, Ibaraki, Japan

Abstract
A supply chain network design problem consists of determining the location of plants and distribution centres, product assignments, transportation and distribution, and inventory levels at facilities, which minimize the total cost, including production, handling, inventory, transportation and distribution costs. Inventory costs are represented as concave functions and facility locations as binary variables, therefore a supply chain network design problem can be formulated as a large mixed integer optimization problem with concave functions. In this paper, the Lagrangian relaxation method is developed for the supply chain network design problem. The Lagrangian relaxation problem formed by relaxing flow conservation constraints is represented as continuous knapsack problems. Finally, for obtaining approximate solutions, a Lagrangian heuristic based on solving linear approximate network flow problems is presented.

Keywords:
supply chain, network design, multi-commodity flow, optimization

1 INTRODUCTION
Recently the global economy has suffered from the deep and long global recession which emerged from the U.S. In order to reduce global costs and adjust the volume of inventory and production, global companies must plan to urgently reconstruct and reduce the scale of their global supply chain networks.

The supply chain network is very large and complicated, and a plan to reconstruct and reduce the scale of a supply chain network is a very difficult problem. Constructing and solving a mathematical model of a supply chain network design are very useful tasks for reconstructing or reducing the network.

A supply chain network design problem consists of determining the location of plants and distribution centres, product assignments, transportation and distribution, and inventory levels in order to minimize the total cost, including production, handling, inventory, transportation and distribution costs. Inventory costs are represented as concave functions and facility locations as binary variables, therefore a supply chain network design problem can be formulated as a large mixed integer optimization problem with concave functions.

In this paper, the Lagrangian relaxation method is developed for the supply chain network design problem. The Lagrangian relaxation problem formed by relaxing multi-commodity flow conservation constraints can be decomposed into subproblems for each node and each product. The subproblems are represented as continuous knapsack problems and concave continuous knapsack problems, and they are easy solvable polynomially. Furthermore, for obtaining approximate solutions, the Lagrangian heuristic based on the solutions of the Lagrangian relaxation problems is developed by solving linear approximation problems.

In the earliest papers, Geoffrion-Graves [1] presents an algorithm based on Benders decomposition to solve a multi-commodity network design problem. Aikens [2], Geoffrion-Powers [3], Vidal-Goetschalckx [4] and others have expanded the model of Geoffrion and Graves.

Integrated supply chain network design problems including locations and inventory costs of distribution centres are developed. Daskin-Couillard-Shen [5] presents an inventory and location model, and Shu-Teo-Shen [6] develops a stochastic transportation and inventory model.

incorporating uncertainty about exchange rates and their risks.

The Lagrangian relaxation method [9] is one of general solution strategies for solving mathematical programming problems that permit us to decompose problems to exploit their particular structure. In the earliest papers, Held-Karp [10] applies a Lagrangian relaxation method and a subgradient procedure to travelling salesman problems. The Lagrangian relaxation method has been applied to a huge number of mathematical problems and yielded excellent results.

2 PROBLEM DESCRIPTION

2.1 Supply chain network design problem

Given potential product plants and potential distribution centres, customers, products and their demands, the supply chain network design problem is to determine product plants and distribution centres, product assignments, transportation and inventory on the network to minimize the total cost, including production, handling, inventory and transportation costs.

Figure 1 and Figure 2 show a supply chain network design problem and a solution of a supply chain network. The supply chain network configuration decision consists of determining which of the production plants and distribution centres to select, which of manufacturing and inventory facilities to be procured from for products at selected plants and centres, and product flows in the network.

Figure 3 shows a network representation of the supply chain network. Product plants, distribution centres and customers are represented as nodes, and transportation is represented as arcs. Product dummy source nodes are defined as origin nodes for product flows.

2.2 Inventory cost

Assuming that a distribution centre orders each product from the upstream product plant using (r, Q) policy with a certain service level, the frequency of orders and the order quantity at each centre are determined by the handling volume (which is the demand of downstream customers).

Let $I_p^i$ denote the inventory holding cost of product $p$ at centre $i$ and $O_p^i$ denote the fixed order cost of product $p$ from the upstream plant at centre $i$. $N_c^i$ is the set of downstream customers of centre $i$. Let $t_p^i$ denote the handling volume of product $p$ at centre $i$, and $n_p^i$ denote the number of orders from the upstream plant at centre $i$.

Then the average order size per order is $t_p^i/n_p^i$, and the average working inventory volume is $t_p^i/2n_p^i$. The total cost of orders and working inventory at centre $i$ is given by

$$O_p^i n_p^i + t_p^i t_p^i / 2n_p^i.$$  

(1)

The optimal value of $n_p^i$ to minimize function (1) is obtained as

$$n_p^i = \sqrt{2I_p^i t_p^i / 2O_p^i}.$$  

(2)

The corresponding total order and working inventory cost $TC_p^i$ of associated node $i$ and product $p$ can be expressed as

$$TC_p^i = \sqrt{2I_p^i O_p^i t_p^i}.$$  

(3)

The volume of safety stock required to ensure that stockouts occur within a certain probability is given by

$$\sigma_p^i L \sum_{j \in N_c^i} \left( \sigma_j^p \right)^2,$$

where $\sigma_p^i$ is the standard deviation of the demand for product $p$ shipped from centre $i$ to customer $j$. The corresponding inventory holding cost of the safety stock is given by

$$\frac{1}{2} \sum_{j \in N_c^i} \left( \sigma_j^p \right)^2.$$  

(4)

Assuming that the ratio of the demand variance to the mean demand is the same value $\beta$ for all downstream customers, i.e.

$$\left( \sigma_j^p \right)^2 / t_j^p = \beta \quad \forall j \in N_c^i.$$

(5)

where $t_j^p$ is the demand for product $p$ shipped from centre $i$ to customer $j$. The total inventory cost $IC_p^i$ can be described as

$$IC_p^i = \left( \sqrt{2} I_p^i O_p^i + t_p^i \sqrt{L \sum_{j \in N_c^i} \left( \sigma_j^p \right)^2} \right).$$

(6)

As described above, the inventory cost can be represented as the square root functions of the handling volume at the centre. Figure 4 illustrates the total inventory cost $IC_p^i$ at centre $i$.

2.3 Problem formulation

Let $G=(N,A)$ be a supply chain network, $N$ be the set of nodes and $A$ be the set of arcs. $N$ consists of the set of product dummy source nodes $N_d$, the set of production plants $N_p$, the set distribution centres $N_c$, and the set of customers $N_c$. $N_{pd}$ is $N_d \cup N_p$, $N^c$ is the set of nodes with the outdegree to other nodes, i.e. $N_+ \cup N_d \cup N_p$, and $N^c$ is the set of nodes with the outdegree to node $i$. $N^c$ is the set of nodes with the in-degree from node $i$. $P$ is the set of products produced and shipped in the supply chain network.
$a_{ij}^p$ is the variable cost associated with node $i$ and an arc from node $i$ to node $j$ of product $p$. If $i \in N_p$ and $j \in N_p$, then $a_{ij}^p$ is the procurement cost of product $p$ at plant $j$ from the product dummy source node $i$. If $i \in N_p$ and $j \in N_p$, then $a_{ij}^p$ is the production cost at plant $i$ and the transportation cost of product $p$ shipped from plant $i$ to centre $j$. If $i \in N_p$ and $j \in N_p$, then $a_{ij}^p$ is the handling cost at centre $i$ and the transportation cost of product $p$ shipped from centre $i$ to customer $j$. $b_{ij}^p$ is a coefficient. $\sqrt{2P_{ij}^pQ_{ij}^p + l_{ij}(a_{ij}^p)}$ associated with the inventory cost of product $p$ at centre $i$, and the inventory cost of product $p$ is proportional to the square root of the handling volume.

$t_i$ is the fixed cost for selecting production plant or distribution centre $i$. $g_i^p$ is the fixed cost for procuring a manufacturing or inventory facility for product $p$ at node $i$. $d_{ij}^p$ is the demand of product $p$ for customer $l$. $C_i$ is the total capacity of production plant or distribution centre $i$. $E_i^p$ is the capacity of a manufacturing or inventory facility at the selected plant and centre $i$ for product $p$.

$y_i$ is the binary decision variable for selecting a plant or centre $i$, which equals 1 if the production plant or distribution centre $i$ is selected, and 0 otherwise. $z_i^p$ is the binary decision variable for procure a facility at plant or centre $i$, which equals 1 if the manufacturing or inventory facility of product $p$ at node $i$ is procured, and 0 otherwise. $x_i^p$ is a continuous decision variable denoting the transportation volume of product $p$ shipped from node $i$ to node $j$ for customer $l$. If $i \in N_p$, then $t_i y_i^p = \sum_{j \in N_p} x_i^p$. $s_i^p$ is a product dummy source node of product $p$.

The supply chain network design problem can be formulated as a mixed integer and nonlinear programming problem $P$ as follows:

\[
\begin{align*}
\text{(P)} & \quad \text{minimize} & & \sum_{i \in N_p} \sum_{j \in N_p} a_{ij}^p x_{ij}^{pl} + \sum_{i \in N_p} \sum_{j \in N_p} b_{ij}^p \sqrt{\sum_{j \in N_p} x_{ij}^{pl}} \\
& & & + \sum_{i \in N_p} \sum_{j \in N_p} f_i y_i + \sum_{j \in N_p} g_i^p z_i^p \\
\text{subject to} & & \sum_{j \in N_p} x_{ij}^{pl} - \sum_{j \in N_p} x_{ij}^{pl} = \begin{cases} -d_{ij}^p & \text{if } s = p^p \\ d_{ij}^p & \text{if } s = l \\ 0 & \text{otherwise} \end{cases} \\
& & & \forall n \in N_p, i \in N_p, c \in P
\end{align*}
\]  

\[
\begin{align*}
\sum_{i \in N_p} \sum_{j \in N_p} x_{ij}^{pl} & \leq C_i y_i \quad \forall i \in N_p \\
\sum_{j \in N_p} x_{ij}^{pl} & \leq E_i^p z_i^p \quad \forall i \in N_p, p \in P
\end{align*}
\]  

\[
\begin{align*}
\sum_{j \in N_p} x_{ij}^{pl} & \leq E_i^p z_i^p \quad \forall i \in N_p, p \in P
\end{align*}
\]  

\[
\begin{align*}
z_i^p & \leq y_i \quad \forall i \in N_p, p \in P \\
0 & \leq x_{ij}^{pl} \leq d_{ij}^p z_i^p \quad \forall i \in N_p, j \in N_p, c \in N_p, p \in P
\end{align*}
\]

3 A LAGRANGIAN RELAXATION METHOD

3.1 A Lagrangian Relaxation Problem

Using the Lagrangian multiplier vector $\nu=(\nu_i^p, \nu_i^p)$ associated with constraints (9) and adding it to the objective function (8), the following Lagrangian relaxation problem $LG$ can be formed.

\[
\begin{align*}
\text{(LG)} & \quad \text{minimize} & & \sum_{i \in N_p} \sum_{j \in N_p} \left( a_{ij}^p x_{ij}^{pl} + \nu_i^p x_{ij}^{pl} \right) + \sum_{i \in N_p} \sum_{j \in N_p} b_{ij}^p \sqrt{\sum_{j \in N_p} x_{ij}^{pl}} \\
& & & + \sum_{i \in N_p} \sum_{j \in N_p} f_i y_i + \sum_{j \in N_p} g_i^p z_i^p \\
\text{subject to} & & \sum_{j \in N_p} x_{ij}^{pl} - \sum_{j \in N_p} x_{ij}^{pl} = \begin{cases} -d_{ij}^p & \text{if } s = p^p \\ d_{ij}^p & \text{if } s = l \\ 0 & \text{otherwise} \end{cases} \\
& & & \forall n \in N_p, i \in N_p, c \in P
\end{align*}
\]

As $LG$ is the relaxation problem of $P$, the optimal value of $LG$ is a lower bound of the optimal value of $P$, and the optimal solution of $LG$ may not be a feasible solution of $P$.

Given the Lagrangian multiplier vector $\nu$, the first term of the objective function (18) is a constant term. Therefore, $LG$ can be decomposed into the following subproblem $LG^v$ for each node $i \in N_p$, and $LG^v$ for each node $i \in N_p$.

\[
\begin{align*}
\text{(LG^v)} & \quad \text{minimize} & & \sum_{i \in N_p} \sum_{j \in N_p} \left( a_{ij}^p x_{ij}^{pl} + \nu_i^p x_{ij}^{pl} \right) + \nu_i^p x_{ij}^{pl} \\
& & & + \sum_{i \in N_p} \sum_{j \in N_p} \left( a_{ij}^p + \nu_i^p - \nu_i^p \right) x_{ij}^{pl} + f_i y_i + \sum_{j \in N_p} g_i^p z_i^p \\
\text{subject to} & & \sum_{j \in N_p} x_{ij}^{pl} - \sum_{j \in N_p} x_{ij}^{pl} = \begin{cases} -d_{ij}^p & \text{if } s = p^p \\ d_{ij}^p & \text{if } s = l \\ 0 & \text{otherwise} \end{cases} \\
& & & \forall n \in N_p, i \in N_p, c \in P
\end{align*}
\]
\[ \sum_{p \in P} \sum_{j \in N_r} x_{pj}^{pl} \leq E_{ij}^{pl} z_{ij}^p \quad \forall p \in P \quad (21) \]

\[ \sum_{m \in N_r, j \in N_r} x_{mi}^{pj} \leq E_{ij}^{pq} z_{ij}^p \quad \forall p \in P \quad (22) \]

\[ z_{ij}^p \leq y_{ij} \quad \forall p \in P \quad (23) \]

\[ 0 \leq x_{nj}^{pl} \leq d_{nj}^p z_{ij}^p \quad \forall l \in N_r, j \in N_r, p \in P \quad (24) \]

\[ 0 \leq x_{nj}^{pl} \leq d_{nj}^p \quad \forall j \in N_r, n \in N_r^+ \quad (25) \]

\[ y_{ij} \in [0,1] \quad \forall p \in P \quad (26) \]

\[ z_{ij}^p \in [0,1] \quad \forall p \in P \quad (27) \]

\[ (L_{G_{ij}^{pl}}) \]

\[
\begin{aligned}
\text{minimize} & \quad \sum_{p \in P} \sum_{j \in N_r} \left( a_{ij}^{pl} + v_{ij}^{pl} - y_{ij}^p \right) x_{ij}^{pl} + \sum_{p \in P} \sum_{j \in N_r} x_{ij}^{pl} \\
& \quad + f_i y_{ij} + \sum_{p \in P} g_{ij}^p z_{ij}^p \\
\text{subject to} & \quad \sum_{p \in P} \sum_{j \in N_r} x_{ij}^{pl} \leq C_i y_{ij} \\
& \quad \sum_{j \in N_r} x_{ij}^{pl} \leq E_{ij}^{pl} z_{ij}^p \\
& \quad 0 \leq x_{nj}^{pl} \leq d_{nj}^p z_{ij}^p \\
& \quad (28) \quad (29) \quad (30) \quad (31) \quad (23), (26), (27) \quad (and (23), (26), (27))
\end{aligned}
\]

\[ \sum_{j \in N_r} x_{ij}^{pl} \text{ can be substituted for } \sum_{j \in N_r} x_{ij}^{pl} \text{ for } i \in N_r. \]

\[ L\bar{G}_{ij}^{pl} \text{ and } L\bar{G}_{ij}^{pl} \text{ can be decomposed into two subproblems in the case of } y=0 \text{ and } y=1. \]

\[ L\bar{G}_{ij}^{pl} \text{ and } L\bar{G}_{ij}^{pl} \text{ can be rewritten as the following problems } L\bar{G}_{ij}^{pl} \text{ and } L\bar{G}_{ij}^{pl}. \]

\[ (L_{G_{ij}^{pl}}) \]

\[ \begin{aligned}
\text{minimize} & \quad \sum_{p \in P} \sum_{j \in N_r} \left( a_{ij}^{pl} + v_{ij}^{pl} - y_{ij}^p \right) x_{ij}^{pl} + \sum_{p \in P} \sum_{j \in N_r} x_{ij}^{pl} \\
& \quad + \sum_{p \in P} \sum_{m \in N_r} \left( a_{ij}^{pl} + v_{ij}^{pl} - y_{ij}^p \right) x_{ij}^{pl} + \sum_{p \in P} g_{ij}^p z_{ij}^p + f_i \\subject{to} & \quad \sum_{p \in P} \sum_{j \in N_r} x_{ij}^{pl} \leq C_i \quad (32) \quad (33) \quad (34) \quad (21), (22), (24), (25), (27) \quad (and (21), (22), (24), (25), (27))
\end{aligned} \]

\[ (L_{G_{ij}^{pl}}) \]

\[ \begin{aligned}
\text{minimize} & \quad \sum_{p \in P} \sum_{j \in N_r} \left( a_{ij}^{pl} + v_{ij}^{pl} - y_{ij}^p \right) x_{ij}^{pl} + \sum_{p \in P} \sum_{j \in N_r} x_{ij}^{pl} \\
& \quad + \sum_{p \in P} g_{ij}^p z_{ij}^p + f_i \\subject{to} & \quad \sum_{p \in P} \sum_{j \in N_r} x_{ij}^{pl} \leq C_i \quad \forall j \in N_r \quad (35) \quad (36) \quad (40) \quad (37) \quad (38) \quad (41) \quad (42) \quad (43) \quad (44) \quad (45) \quad (46) \quad (and (43))
\end{aligned} \]

Furthermore, using the nonnegative Lagrangian multiplier \( u_i \) associated with capacity constraints (33) and (35), and adding them to the objective functions (32) and (34), the following Lagrangian relaxation problems \( L_{G_{ij}^{pl}} \) and \( L_{G_{ij}^{pl}} \) can be formed.

\[ (L_{G_{ij}^{pl}}) \]

\[ \begin{aligned}
\text{minimize} & \quad \sum_{p \in P} \sum_{j \in N_r} \sum_{i \in N_r} \left( a_{ij}^{pl} + v_{ij}^{pl} - y_{ij}^p + u_i \right) x_{ij}^{pl} + + \sum_{p \in P} \sum_{j \in N_r} \sum_{i \in N_r} \left( a_{ij}^{pl} + v_{ij}^{pl} - y_{ij}^p \right) x_{ij}^{pl} + + \sum_{p \in P} g_{ij}^p z_{ij}^p - C_i u_j + f_i \\subject{to} & \quad (21), (22), (24), (25), (27) \quad (39) \quad (40) \quad (41) \quad (42) \quad (43) \quad (44) \quad (45) \quad (46) \quad (and (43))
\end{aligned} \]
\[
\phi_p^L = \text{minimize} \sum_{j \in N_c} \sum_{i \in N_c} \left( a_{ij}^P + v_{ij}^P - v_{ij}^P + u_i \right) x_{ij}^P + \sum_{m \in N_c} \sum_{i \in N_c} \left( a_{mi}^P + v_{mi}^P - v_{mi}^P + g_i^P \right)
\]

subject to \[
\sum_{i \in N_c} x_{ij}^P \leq E_j^P \quad \forall j \in N_c \quad (48)
\]
\[
\sum_{m \in N_c} \sum_{i \in N_c} x_{mi}^P \leq E_m^P \quad \forall m \in N_c \quad (49)
\]
\[
0 \leq x_{ij}^P \leq d_j^P \quad \forall l \in N_c, j \in N_c^l \quad (50)
\]
\[
0 \leq x_{mi}^P \leq d_m^P \quad \forall l \in N_c, n \in N_c^l \quad (51)
\]

\[LG_{p \text{trct}}\] is a concave continuous knapsack problem, and can be solved in polynomial time [11]. The objective function (52) is concave, therefore the optimal solution is one of the extreme points on the function. Figure 5 illustrates the extreme points on the function.

Let \( h = a_{ij}^P + v_{ij}^P - v_{ij}^P + u_i \), and permutation \( m \) satisfies the following equation.
\[
h_m \leq h_{m+1} \leq \cdots \leq h_{m+n} \quad (55)
\]
\[
H_q = \sum_{k=1}^{q} h_m d_m + b_m \sqrt{\sum_{k=1}^{q} d_m^p} \quad \forall q < \tilde{q} \quad (56)
\]
\[
H_q = \sum_{k=1}^{q-1} h_m d_m + \left( E_P - \sum_{k=1}^{q-1} d_m \right) h_m + b_m \sqrt{E_P^m} \quad (57)
\]

where \( \tilde{q} \) is the minimum index \( q \) which satisfies \( \sum_{k=1}^{q} d_m^p \geq E_P^m \).

Let \( H = \min_{1 \leq q \leq \tilde{q}} H_q \). If \( H \leq 0 \), then the optimal solution is \( x = 0 \), or otherwise as follows:
\[
x_{lm} = \begin{cases} 
  d_m^p & k = 1, \ldots, \tilde{q} - 1 \\
  E_P^m - \sum_{k=1}^{\tilde{q}-1} d_m^p & k = \tilde{q} \\
  0 & k > \tilde{q}.
\end{cases}
\]

As mentioned above, the Lagrangian relaxation problem \( LG \) can be solved polynomially, and the lower bound \( LB \) of \( P \) can be obtained as follows:
\[
LB = \sum_{i \in N_p} \min \left\{ 0, \sum_{p \in P} \left[ \phi_{ip}^L - C_i u_i + f_j \right] \right\} + \sum_{p \in P} \left( \sum_{i \in N_p} \left[ \phi_{ip}^L - C_i u_i + f_j \right] \right) \quad (59)
\]

3.2 A SUBGRADIENT METHOD

For choosing the appropriate value of \( u \), a binary search is applied for each \( LG_{p \text{trct}} \) and \( LG_{d \text{trct}} \). The range of the optimal value of \( u_i \) is from \(-\min(a_{ij}^P + v_{ij}^P - v_{ij}^P)\). Given each \( u_i \) at the binary search, \( LG_{p \text{trct}} \) and \( LG_{d \text{trct}} \) are solved. And the optimal value of \( u_i \) such as minimizing the optimal value of \( LG_{p \text{trct}} \) and \( LG_{d \text{trct}} \) is found.

For choosing the appropriate value of \( v \), approximately, the subgradient optimization procedure is applied. This is an iterative procedure, which uses the current multipliers \( v \), the current lower bound and an upper bound, in order to compute the new multipliers \( v \) used in the next iteration. The subgradient \( w \) of \( v \) at the current solution \( x \) of \( LG \) can be defined as follows:
\[
\left\{ \begin{array}{ll}
  -d_m^p - \sum_{i \in N_c^l} r_{ij}^p + \sum_{i \in N_c^l} r_{ij}^p & \text{if} \ n = s^P \\
  -d_m^p - \sum_{i \in N_c} r_{ij}^P + \sum_{i \in N_c} r_{ij}^P & \text{if} \ n = l \\
  -\sum_{i \in N_c^l} r_{ij}^p + \sum_{i \in N_c^l} r_{ij}^p & \text{otherwise}
\end{array} \right. \forall n \in N, l \in N_c, p \in P \quad (60)
\]

By using a step size \( s' \) in iteration \( t \), the new set of multipliers are given by
\[
v_{n}^{t+1} := v_{n}^{t} + s' w_{n}^{t} \quad \forall n \in N, l \in N_c, p \in P \quad (61)
\]

The step size \( s' \) is selected according to
\[
s' = \rho(\text{upper bound} - \text{LB})/|w|^2 \quad (62)
\]

where \( \rho \) is a scalar, which is initially equal to 1 and is reduced every certain iteration number.

4 LAGRANGIAN HEURISTIC

A Lagrangian heuristic is an approximate solution method, and derives feasible solutions from Lagrangian relaxation solutions. The optimal solutions \( \bar{x}, \bar{x}, \bar{y} \) of \( LG \) and lower bound \( LB \) of \( P \) are obtained by solving \( LG \). But \( \bar{x} \) may not be feasible for \( P \), due to relaxation of flow conservation.
constraints. The Lagrangian heuristic is presented based on \( \bar{x} \), by using a linear approximation near \( \bar{x} \).

The objective function of \( P \) involves nonlinear functions. \( P \) is not a linear programming problem, therefore it is hard to solve \( P \) by a mathematical programming solver such as CPLEX. Therefore the linear approximation of the square root function near \( \bar{x} \) is used as follows:

\[
\sqrt{\sum_{j \in J} x_{ij}^p} = \frac{1}{2A(\bar{x})} \sum_{j \in J} x_{ij}^p - \frac{1}{2} \frac{d}{A(\bar{x})^2} \sum_{j \in J} x_{ij}^p
\]

where \( A(\bar{x}) = \sum_{j \in J} x_{ij}^p \).

The linear approximation problem near \( \bar{x} \) at \( \bar{z} \) and \( \bar{y} \) of \( P \) is formed as the following linear programming problem \( LA(\bar{x}) \).

( LA(\bar{x}) )

\[
\text{minimize} \quad \sum_{i \in N} \sum_{p \in P} \sum_{j \in J} a_{ij}^p x_{ij}^p + \sum_{i \in N} \sum_{p \in P} \sum_{j \in J} b_{ij}^p \sqrt{\sum_{j \in J} x_{ij}^p} - \sum_{j \in J} b_{ij}^p A(\bar{x}) \frac{d}{2} \sum_{p \in P} f_{ij}^p + \sum_{p \in P} g_{ij}^p \frac{d}{2} \sum_{p \in P} f_{ij}^p + g_{ij}^p \frac{d}{2} \sum_{p \in P} f_{ij}^p + g_{ij}^p
\]

\[
\sum_{p \in P} \sum_{j \in J} x_{ij}^p \leq C_{ij}, \quad \forall i \in N_{pd}
\]

\[
\sum_{j \in J} x_{ij}^p \leq E_{ij}, \quad \forall i \in N_{pd}, p \in P
\]

\[
\sum_{i \in N} x_{ij}^p \leq E_{ij}, \quad \forall i \in N_{pd}, p \in P
\]

\[
0 \leq x_{ij}^p \leq d_{ij}, \quad \forall i \in N_{pd}, j \in N_{pd}, p \in P
\]

\[
0 \leq x_{ij}^p \leq d_{ij}, \quad \forall i \in N_{pd}, j \in N_{pd}, n \in N_{pd}, p \in P
\]

\( LA(\bar{x}) \) can be solved by a mathematical programming solver. \( \bar{x} \) is an initial solution for \( LA(\bar{x}) \). Let the solution of \( LA(\bar{x}) \) denote \( \bar{x}^1 \), and \( \bar{x}^k \) denote the solution at the kth iteration. \( LA(\bar{x}) \) is solved iteratively until \( \bar{x}^k \) converges. If \( LA(\bar{x}) \) is not feasible, a feasible solution of \( P \) cannot be obtained from \( \bar{x} \). In the subgradient method, various Lagrangian relaxation solutions are obtained at iterations, therefore various initial feasible solutions and various convergent feasible solutions can be obtained.

The whole algorithm of the Lagrangian relaxation method for the supply chain network design problem is described as follows:

[Step1] Given \( v, t=0 \), \( ite_{\text{max}} \)=the iteration of maximum numbers, \( LB=0, UB=\infty \).

[Step2] Solve \( LG \) with \( v \) and obtain \( \bar{x}, \bar{z}, \bar{y} \).

[2-1] Set \( u \) at the binary search, and solve \( LG^{p/2} \) and \( LG^{p/2} \).

[2-2] Find the optimal value of \( u \) and obtain \( LB^t \).

[2-3] If \( LB^t > LB \), then \( LB= LB^t \).

[Step3] Solve \( LA(\bar{x}) \) at \( \bar{z} \) and \( \bar{y} \).

[3-1] If \( LA(\bar{x}) \) is feasible, then obtain the convergent feasible solution and upper bound \( UB \).

[3-2] If \( UB < UB \), then \( UB = UB \).

[Step4] Update \( v \) by the subgradient method.

[Step5] \( t=t+1 \). If \( t>ite_{\text{max}} \), then stop, else go to Step2.

5 CONCLUSION

In this paper, the Lagrangian relaxation method is developed for the supply chain network design problem. The supply chain network design problem is formulated as a mixed integer and nonlinear programming problem, and the Lagrangian relaxation problem formed by relaxing multi-commodity flow conservation constraints can be decomposed into subproblems for each node and each product. The subproblems are represented as continuous knapsack problems, and are solvable polynomially. Finally, for obtaining approximate solutions, the Lagrangian heuristic based on the solutions of the Lagrangian relaxation problems is developed by solving linear approximation problems.

Numerical experiments to evaluate the effectiveness of the Lagrangian relaxation method have not been presented yet. This research is underway to evaluate by numerical experiments and apply to real world problems.

References


