A Product-to-Plant Allocation Problem in Logistics Network Design

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Abstract
Recently, M&A between companies that produce similar products have been carried out quite often with the intention of improving efficiency and economies of scale. In this paper, a case study is conducted on an M&A model for a Japanese tire company. This model can be formulated as a large-scale mixed integer programming problem. A new formulation and approximate solution method with a tabu search are developed. By using our method, good approximate solutions can be easily obtained within a reasonable computation time using PC. Finally, an analysis using actual case data for the Japanese tire company is carried out and a good product-to-plant allocation is obtained. From our results, it is found that the total cost is improved by 16.85%, the equivalent of $5 million. Our solution is feasible and satisfactory for business managers of the company, and our study can contribute to allocation planning for many other companies.

Keywords:
Logistics design, Product-to-plant allocation, Optimization

1 INTRODUCTION

Recently, M&A between companies that produce similar products have been carried out quite often with the intention of improving efficiency and economies of scale. This reorganization brings about a large-scale product-to-plant allocation/reallocation problem to minimize logistics and production costs in the framework of a new logistics network. A product-to-plant allocation problem is concerned with tactical level decision-making to allocate products to manufacturing plants.

Given plants and production capacities, markets, and demands, a product-to-plant allocation problem can be formulated as a kind of allocation or assignment problem with side constraints. This problem is known as a generalized assignment, a capacitated facility location or a logistics network design problem, depending on the type of side constraints. Jordan and Graves [1] introduced the concept of chaining in the context of the automotive industry, and they propose that two plants are considered linked if they share a product. Hodgson et al. [2] presented the decision support system for furniture production, which includes assigning products to kilns. Peters and McGinnis [3] modeled the product allocation problem for single stage electronic assembly systems. Inman and Gonsalvez [4] presented the product-to-plant allocation problem actually faced in the automotive industry and provide an optimization-based decision support approach. Alden et al. [5] presented a product-to-plant allocation process and a decision support computer tool to assess the cost and time to allocate a new product to a manufacturing plant. Chaudhuri and Singh [6] proposed a model to maximize profit, by considering capacity addition costs, assembly line retooling costs, transportation costs and fully built products.

In this paper, a case study is conducted on a major Japanese tire company that produces tires for domestic and overseas markets. This company recently acquired another fifth largest tire company to become the second largest in Japan. Production capacities of plants for product families and full container load constraints are considered as side constraints in our case study. The optimal product-to-plant allocation and the optimal container port assignment for overseas products are found in the problem.

This problem can be formulated as a large-scale mixed integer programming model. In our case study, as we deal with more than 1000 products and more than 5000 integer variables, it is difficult to solve optimally using optimization software and to solve directly using general solution methods such as the branch-and-bound method. Therefore, a new formulation and approximate solution method combining a tabu search and a random multi-start technique should be developed. By using our algorithm, a good approximate solution can be obtained within a reasonable computation time using PC. The solution is feasible and satisfactory for business managers of the company, and our study can contribute to allocation planning for many other companies.

2 MATHEMATIACL FORMULATION

The company in our case study imports materials from overseas and produces a great number of products at several domestic manufacturing plants. Their products are divided into several product families and they are sold in domestic and overseas markets. Plants have production lines with production capacities for each product family. The products for foreign sales are exported via container ports in Japan to export partners by full container load. The products for domestic markets are classified with retail products via distribution centers and OEM products sold to auto manufacturers. Figure 1 shows the transportation network of the company in this case study. The broken line in this figure shows the scope of our study.

As the production costs do not vary by manufacturing plant location in our case study, we only take transportation costs into consideration. As materials are imported to the nearest ports of the plants by oil tankers and bulk carriers, and transportation costs of materials are inexpensive relative to those of products, transportation costs for materials are not considered in our model. For those reasons stated above, transportation costs of products from plants to distribution centers, auto manufacturers and container ports are dealt with in this paper.

Another important issue relates to the full container load constraint. This constraint means that products must be shipped in full container loads from container ports to export partners. As the shipping cost of a marine container is expensive and depends on the number of containers, the number of containers must be minimized and should be appropriate for the volume of products for foreign sales

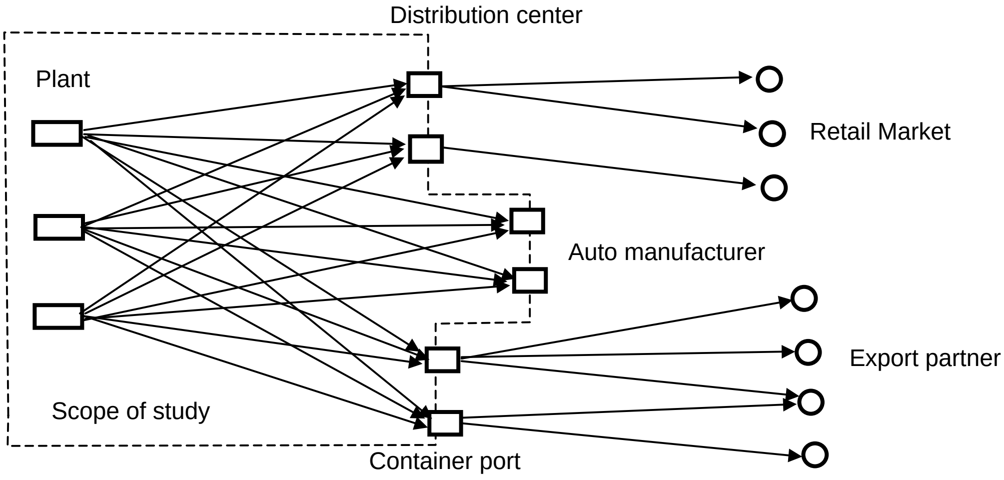

Figure 1: Transportation network

shipped to each export partner. Therefore, the number of containers for each export partner is preliminarily determined by the volume shipped to each export partner. Containers can be shipped via different container ports in Japan to an export partner.

In formulating our product-to-plant allocation problem, several conditions are assumed. These assumptions are as follows:

- A number of products, manufacturing plants, distribution centers, auto manufacturers and container ports are given.
- For each product, demands for domestic markets and export partners are given.
- Products are classified with product families.
- Each manufacturing plant has a production capacity for each product family.
- Each product is produced at one plant.
- Products for domestic markets are sold via distribution centers or to auto manufacturers.
- Transportation costs per unit from plants to distribution centers, auto manufacturers and container ports are given.
- The objective function is the sum of transportation costs from plants to distribution centers, auto manufacturers and container ports, and should be minimized.
- Products shipped to partners are exported via container ports in full container loads.
- The capacity of a marine container is given.
- The number of marine containers shipped to each export partner is given.

The product-to-plant allocation problem is formulated as follows:

minimize $\sum_{m\in M}\sum_{k\in K_m}\sum_{i\in I_m}\sum_{j\in J} a_{ij}^k y_{ij}^k$ (1)

subject to

$$\sum_{j\in J} y_{ij}^k = b^k x_i^k \qquad i\in I_m, k\in K_m, m\in M \qquad (2)$$

$$\sum_{i\in I_m} y_{ij}^k = d_j^k \qquad j\in J_1, k\in K_m, m\in M \qquad (3)$$

$$\sum_{i\in I_m} y_{ij}^k = \sum_{l\in L} s_{jl}^k \qquad j\in J_2, k\in K_m, m\in M \qquad (4)$$

$$\sum_{j\in J_2} s_{jl}^k = d_l^k \qquad l\in L, k\in K_m, m\in M \qquad (5)$$

$$\sum_{k\in K_m} b^k x_i^k \le c_i^m \qquad i\in I_m, m\in M \qquad (6)$$

$$\sum_{i\in I_m} x_i^k = 1 \qquad k\in k_m, m\in M \qquad (7)$$

$$\sum_{m\in M}\sum_{k\in K_m} s_{jl}^k \le C z_{jl} \qquad j\in J_2, l\in L \qquad (8)$$

$$\sum_{j\in J_2} z_{jl} \le \left\lceil \frac{\sum_{m\in M}\sum_{k\in K_m} d_l^k}{C} \right\rceil \qquad l\in L \qquad (9)$$

$$y_{ij}^k \ge 0 \qquad i\in I_m, j\in J, k\in K_m, m\in M \qquad (10)$$

$$s_{jl}^k \ge 0 \qquad j\in J_2, l\in L, k\in K_m, m\in M \qquad (11)$$

$$x_i^k \in \{0,1\} \qquad i\in I_m, k\in K_m, m\in M \qquad (12)$$

$$z_{jl} \in Z^+ \qquad j\in J_2, l\in L \qquad (13)$$

where

M : Set of product families

K_m : Set of products in product family m

I_m : Set of plants, where product family m is produced

J_1 : Set of domestic markets

J_2 : Set of container ports

J : $J_1 \cup J_2$

L : Set of export partners

a_{ij}^k : transportation cost per unit from plant i to j ($\in J \cup L$) for product k

d_j^k : Demand of product k for j ($\in J$)

b^k : Supply of product k ($= \sum_{l\in L\cap J_1} d_l^k$)

c_i^m : Production capacity of plant i for product family m

C : Capacity of a marine container

x_i^k : Product-to-plant allocation variable; 1 if product k is allocated to plant i, 0 otherwise

y_{ij}^k : Flow variable; transportation volume of product k from plant i to j ($\in J \cup L$)

s_{jl}^k : Flow variable; export volume of product *k* from port *i* to export partner *l*

z_{jl} : Container number variable; the number of containers exported from port *j* to export partner *l*

Z^+ : Set of nonnegative integers

Equation (1) is the objective function of the total transportation costs and is to be minimized. Constraints (2) ensure that the supply and the transportation volume of product *k* are met at plant *i*. Constraints (3) ensure that the transportation volume and the demand of product *k* are met at domestic market *j*. Constraints (4) represent that the transportation volume of product *k* from plants to port *j* is equal to the export volume to export partners. Constraints (5) ensure that the demand and the export volume shipped to export partner *l* of product *k* are met. Constraints (6) ensure that the production volume of product *k* is less than or equal to the production capacity for product family *m* at plant *i*. Constraints (7) represent that the sum of product-to-plant allocation variables is equal to 1, and each product is allocated to one plant. Constraints (8) ensure that the export volume shipped to each export partner at port *j* is less than or equal to the capacity of a marine container multiplied by the number of containers. Constraints (9) represent the relationship between the export volume and the number of containers shipped to each export partner. Constraints (8) and (9) represent full container load conditions. Constraints (10) and (11) ensure the nonnegativity of variables, constraints (12) force binary variables, and constraints (13) force integral variables.

The problem, except constraints (8) and (9), is to allocate products to plants under production capacities. This problem is reduced to a kind of allocation or assignment problem with capacity constraints. On the other hand, the problem considering constraints (8) and (9) is a complex problem including how many marine containers should be exported to each export partner at each port and which products should be loaded to each container under the marine container capacity.

This problem is formulated as a mixed integer problem. If the number of products and markets/partners is small, this problem can be solved using mathematical optimization software. Unfortunately it is impossible to solve the problem with more than 5000 integer variables in our case study within a reasonable computation time by using commercially available software. In the meantime, not the exact solution but a feasible and satisfactory approximate solution is needed in practice. Thus, an approximated solution method combining a tabu search and a random start technique is proposed.

3 A SOLUTION METHOD

The product-to-plant allocation problem can be divided into two problems; 1) allocation problems such as which products should be produced at plants under production capacities, 2) transportation problems related to the transport of products from plants to domestic markets and container exports. If a product-to-plant allocation is determined, the transportation problem to domestic markets becomes an easy solvable problem, which calculates the transportation cost from an allocated plant to domestic markets. On the other hand, the transportation problem of overseas products can be viewed as the combining problem; 1) how many containers should be exported to each export partner at each port, 2) which products should be loaded to a container at each port. When containers are considered as facilities, this problem is reduced to a kind of capacitated facility location problem without facility costs. The large capacitated facility location problem is hard to solve optimally. But if the number of

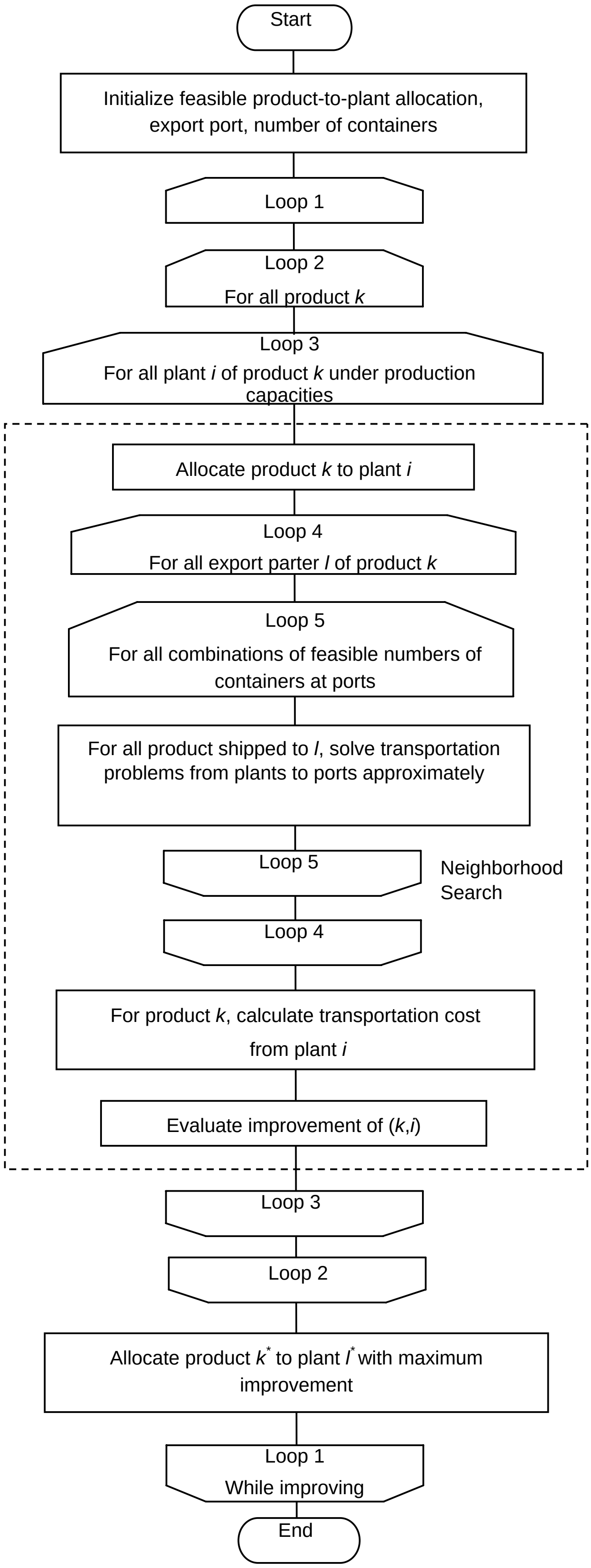

Figure 2: Local search

containers at each port is given, this problem is reduced to a simple Hitchcock type transportation problem and can be solved easily.

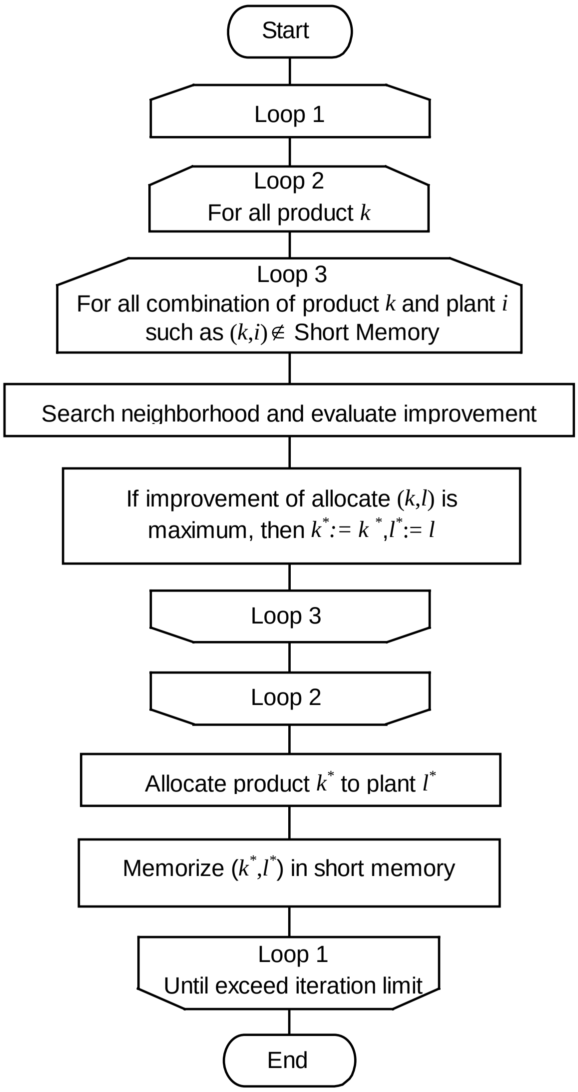

Figure 3: Tabu search

Based on the above understanding, product-to-plant allocation variables are considered as decision variables. By applying the product-to-plant allocation variables to a tabu search with a random start technique, these variables are changed and approximate solutions are searched. After these variables are changed and determined, the transportation costs for domestic markets are calculated. And ports, at which containers are exported to each export partner, and the numbers of containers are determined approximately. Furthermore Hitchcock type transportation problems are solved and the transportation costs for overseas products are obtained. Then, the total cost can be evaluated for the product-to-plant allocation change.

3.1 Local search and evaluation of move

Figure 2 shows the flow chart for the local search. The local search is based on searching neighborhoods which change a product-to-plant allocation variable at the current solution. This method consists of 1) calculating the difference of the total costs between the current solution and a neighborhood solution, 2) finding the solution with maximum improvement in neighborhoods, 3) moving the solution iteratively while an improvement solution is found.

Loop 1 in Figure 2 is the main loop for finding improvement solutions. Loops 2 and 3 are changes of a product-to-plant allocation. To change a product-to-plant allocation from some plant to another plant is called a 'move'. Loops 4 and

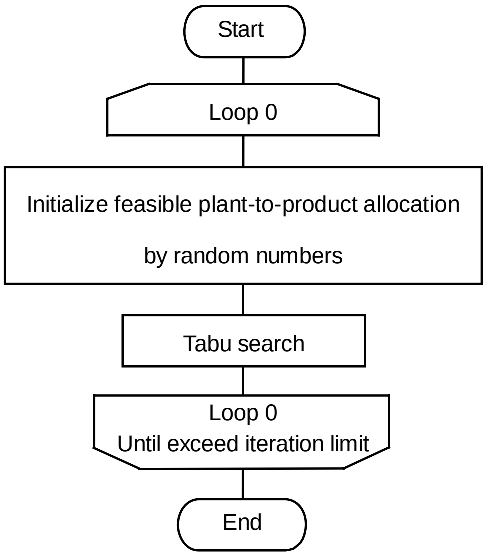

Figure 4: Random multi-start tabu search

5 are changes of container ports and the numbers of containers for export partners.

After a move, improvement of this move needs to be evaluated. Improvement of the move for domestic markets can be easily evaluated by calculating the transportation cost of products shipped from a new allocated plant. For the overseas products, to evaluate improvement of the move is complicated. When changing a product-to-plant allocation of product k, the transportation costs of product k from the plant to container ports change. Then, the total transportation cost associated with all export partners to whom product k is exported, also changes. By changing this cost, if ports shipped to these export partners and/or the numbers of containers are changed appropriately, the total transportation cost may be reduced. Consequently the move of a product-to-plant allocation may cause ports for export partners to change. Furthermore, the transportation costs of all products shipped to these export partners may be changed.

When changing a product-to-plant allocation of product k, the transportation costs associated with product k must be recalculated. And the ports and the number of containers must be found, such that the total transportation cost associated with all export partners of product k is minimized. As a large number of moves are iterated in the local search, it takes a large computation time to find an exact minimum transportation cost for each move. Therefore the minimum transportation cost is calculated approximately as follows;

[Step 1] For all export partners of product k, for all t=1 to *min* (the number of containers shipped to export partner l, the number of ports), do Step 2–8;

[Step 2] For all combinations of ports such as the total number of containers = t, do Step 3–8;

[Step 3] For all port j, z_{jl} :=1.

[Step 4] For all products exported to export partner l, calculate the minimum transportation cost from the allocated plant to ports such as z_{jl} :=1.

[Step 5] In ascending order of the transportation costs of products, do Step 6–8;

[Step 6] While the slack variable of constraint (8) is positive, transport to port j with the minimum cost from the allocated plant.

[Step 7] If the slack variable of constraint (8) is 0 and the slack variable of constraint (9) is positive, then $z_{jl} := z_{jl} + 1$.

[Step 8] If all demand is not transported, then go to Step 6.

In Steps 1 and 2, the possible combinations of ports for all export partners associated with product *k* are set. In Step 3, the initial numbers of containers at ports are given. In Step 5, products are set in ascending order corresponding to the minimum transportation costs. In Step 6 and 7, at first the port with the minimum cost is transported under the marine container capacity. Next, if residual containers exist, then the number of containers is added.

3.2 Tabu search and random multi-Start

As a local search searches only a better solution than the current solution, the convergence solution is one of local optimums. A tabu search [7] is one of meta-heuristic optimization methods, belonging to the class of local search techniques, and it enhances the performance of a local search by using memory structures. Using a short-term memory, the restricted neighborhoods are searched. The current solution can be escaped from the local optimum and the method can search solutions in a wide feasible region.

In our tabu search, a short-term memory is used for prohibiting the move of combination product *k* and new allocated plant *i*. In the iteration, if product *k* is allocated to plant *i*, then the combination (*k*, *i*) is stored in the short-term memory for an assigned time period. While (*k*, *i*) is included in the short-term memory, the move of product *k* and plant *i* is prohibited. In these restricted neighborhoods, the best improvement move is searched and the current solution moves this solution iteratively. Figure 3 shows the flow chart of the tabu search.

In addition, the tabu search with a random multi-start technique is applied. Uniform random numbers give problems initial product-to-plant allocations periodically. This method can search solutions in a wider feasible region than that of a single-start version, and it can find better solutions. Figure 4 shows the flow chart of the random multi-start tabu search.

4 A CASE STUDY

One of the major tire companies in Japan is analyzed as our case study. This company recently acquired another fifth largest tire company to become the second largest in Japan and restructured its product-to-plant allocations for efficient management. This company has four major manufacturing plants as shown in Figure 5, and it produces three major product families, which are tires for auto truck-buses, light trucks and personal cars. The number of products is over 1100. The company provides OEM products to about 30 auto manufacturers and eight distribution centers shown in figure 7 as retail markets. Products are exported from four container ports shown in figure 6 to about 100 countries. The locations of the major export partners are shown in figure 8.

The computational conditions for this case study are as follows;

a) The number of multi-random starts: 5

b) The number of tabu search iterations: 200

c) Memory period in the short-term memory:

$50,100,150,200 \times (1+0.5 \times \text{uniform random number})$

d) Computer language: Visual Basic Ver.6.0

e) PC: CPU Pentium4 3.2Mz, Memory 1Gbyte

Figure 9 shows the change in the total transportation cost according to memory periods. The total transportation cost of the new product-to-plant allocation found by the

Figure 5: Plant location

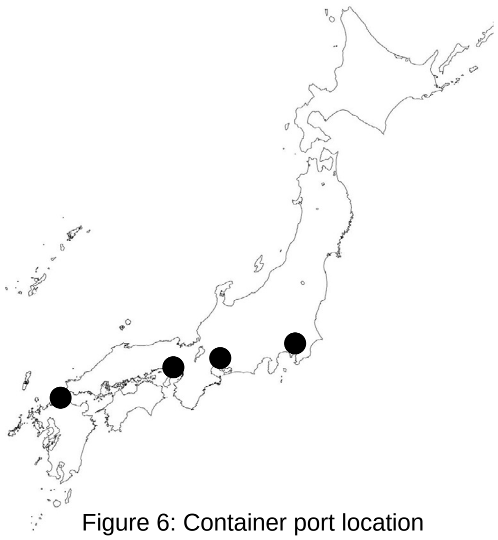

Figure 6: Container port location

Figure 7: Locations of auto manufactures and distribution centers

Figure 8: Major export partner

proposed method can be reduced by 16.85% compared to the current cost. This percentage is equivalent to about 5 million dollars per year. At the best solution, the number of changed product-to-plant allocations made up about 60% (700 products) of the total products. The computation time was about 5 hours.

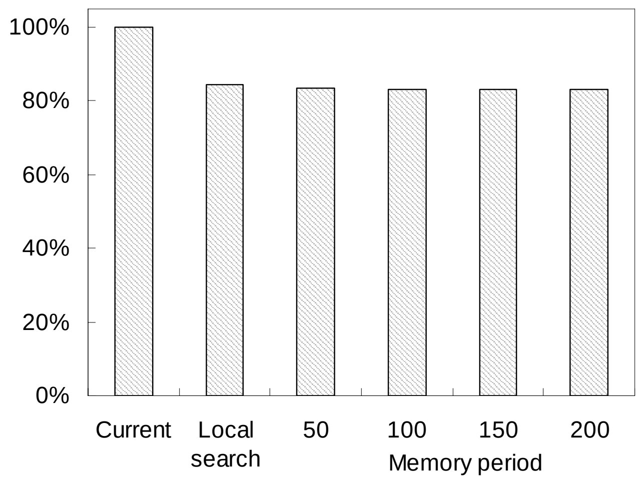

Figure 9: Change in total costs according to memory period

5 CONCLUSION

In this paper, a case study on a major Japanese tire company that produces domestic and overseas products was conducted. Production capacities of product families and full container loads are considered side constraints. This model can be formulated as a large-scale mixed integer programming problem with more than 5000 integer variables. An approximate solution method combining a tabu search and a random start technique was developed. By using our solution method, a good heuristic solution could be obtained within a reasonable computation time using PC.

Finally, actual case data of the Japanese tire company was analyzed, and a good new product-to-plant allocation was obtained. The total cost of the new allocation was reduced by 16.85%, the equivalent of $5 million per year. The number of changed product-to-plant allocations made up about 60% (700 products) of the total products. Our solution is feasible and satisfactory for business managers of the company and can contribute to allocation planning for many companies.

6 ACKNOWLEDGEMENTS

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