

A MODEL FOR THE OPTIMAL NUMBER AND LOCATIONS OF PUBLIC DISTRIBUTION CENTERS

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ABSTRACT

Distribution centers managed by public authorities have been designed to achieve global optimization for efficient logistics and maintain the urban environment in Japan. From the standpoint of social logistics, global optimization should be pursued considering both the environmental aspects and the efficiency of business logistics. In this study, we propose a mathematical programming model for obtaining the optimal number and locations of public distribution centers. This model aims at reducing truck CO₂ emissions while minimizing logistics costs. Our model is applied to the Tokyo metropolitan area and its validity is evaluated. In this way, an appropriate policy for public distribution centers could be proposed.

1. INTRODUCTION

Supply chain management and cooperative deliveries are used by many companies these days in order to reduce physical distribution costs. Moreover, distribution centers managed by public authorities in Japan have been designed to achieve global optimization for efficient logistics and maintain the urban environment. These are attempts toward the global optimization of logistics. A survey about the utilization of public distribution centers by firms indicated that around 60% of participants wanted to make use of them. According to the survey, most of the firms were interested in the availability of land and the use charge.

In this study, we propose a mathematical programming model for obtaining the optimal number and locations of public distribution centers in the Tokyo metropolitan area.

When considering the impact on the environment, we should minimize the sum of the vehicle-kilometer (the number of vehicles \times delivery distances) in the metropolitan area to reduce truck CO₂ emissions. To that end, the number of distribution centers should be increased so as to decrease the delivery distances within the area. However, many companies tend to reduce the number of distribution centers to minimize stock volumes and costs.

From the standpoint of social logistics, global optimization should be pursued considering both the environmental aspects and the efficiency of business logistics. We have to reduce the amount of CO₂ emitted by trucks while minimizing logistics costs. Transportation costs from supply points to distribution centers, delivery costs from distribution centers to customers, and operating costs in the centers are included in logistics costs. Solutions minimizing total logistics costs are reconsidered in terms of truck CO₂ emissions. Finally, this model is applied to the Tokyo metropolitan area, and the optimal number and locations of distribution centers is calculated. In addition, the validity of our model is evaluated.

2. MODEL AND ALGORITHM

In reality, most companies do their best to reduce logistics costs rather than the sum of vehicle-kilometers or CO₂ emissions. Therefore we first consider the problem of minimizing logistics costs, namely, transportation, delivery and operating costs. Next, we evaluate solutions to the cost-minimizing problem by measuring vehicle-kilometer and CO₂ emissions and reconsider the number of centers and center locations by both logistics costs and CO₂ emissions.

We make the following set of assumptions to formulate our model.

1. Supply points, demand points and candidate distribution center points are given as a set of nodes.
2. Supply points are located at the entrance nodes from outside the Tokyo metropolitan area and in nodes with main sources within that area.
3. Distribution centers can handle unlimited amounts of any kind of the commodities.

4. The amounts of supply and demand by goods at all nodes are given.
5. Any kinds of goods from supply points are transported to demand points via the distribution center.
6. Operating costs at distribution centers are increased nonlinearly by the amounts of handling goods.

Moreover, we define the following notation.

K : the commodity set;

S : the supply node set;

F : the candidate distribution center set;

D : the demand node set;

$x_{ij}^k=1$ if we transport all demands of commodity k from supply node i to center j , otherwise $x_{ij}^k=0$;

$y_j=1$ if we locate at candidate center j , otherwise $y_j=0$;

$z_{jl}^k=1$ if we deliver all demands of commodity k from center j to demand node l , otherwise $z_{jl}^k=0$;

d_{il}^k : demand of commodity k from supply node i to demand node l ;

c_{ij}^k : transportation cost from supply node i to center j ;

e_{jl}^k : delivery cost from center j to demand node l ;

f_j : the non-linear operating cost function with respect to center j ;

b_j : the amount of demand handled at center j ;

u : number of public distribution centers.

The problem is to determine the number and locations of public distribution centers to minimize total logistics costs. Using the above-mentioned notation, we can formulate the integer non-linear mathematical programming model as follows:

$$\text{minimize } \sum_{i \in S} \sum_{j \in F} \sum_{k \in K} c_{ij}^k x_{ij}^k + \sum_{j \in F} \sum_{l \in D} \sum_{k \in K} e_{jl}^k z_{jl}^k + \sum_{j \in F} f_j(b_j) y_j \quad (1)$$

$$\text{subject to } \sum_{i \in S} x_{ij}^k = 1 \quad k \in K, j \in F \quad (2) \quad \sum_{l \in D} z_{jl}^k = 1 \quad k \in K, j \in F \quad (3)$$

$$x_{ij}^k \leq y_j \quad k \in K, i \in S, j \in F \quad (4) \quad z_{jl}^k \leq y_j \quad k \in K, l \in D, j \in F \quad (5)$$

$$\sum_{l \in D} \sum_{i \in S} \sum_{k \in K} d_{il}^k x_{ij}^k = \sum_{i \in S} \sum_{l \in D} \sum_{k \in K} d_{il}^k z_{jl}^k = b_j \quad j \in F \quad (6) \quad \sum_{j \in F} y_j = u \quad (7)$$

$$x_{ij}^k \in \{0,1\} \quad k \in K, j \in F, i \in S \quad (8) \quad z_{jl}^k \in \{0,1\} \quad k \in K, l \in D, j \in F \quad (9)$$

$$y_j \in \{0,1\} \quad j \in F \quad (10) \quad u \in \text{integer} \quad (11)$$

Equation (1) is the objective function, which should be minimized. The first term in this equation represents the transportation costs from supply points to the distribution centers. The second term is the delivery costs from distribution centers to demand points. The third term is the operating cost at these centers, which depends non-linearly on the amount of handled goods.

Finding an optimal solution to this problem is difficult because of its integrality and non-linearity. Therefore, we adopted a heuristic algorithm to obtain the solution.

Given the number of public distribution centers and their temporal locations, each demand node should be assigned to the center node with the lowest distribution cost, since we assume that distribution centers are incapacitated. A created set of nodes assigned to the same center forms the new territory of this center. We expect the new center serving each demand node in the territory to be located at the optimal node, minimizing the total cost in each territory. This problem is known as the 1-median problem. We can find these new locations by a simple enumeration method. Then, we replace current locations by new locations. While the total cost is reduced, the procedure is repeated to find new territories and new center locations and replace them with new ones. This method is a so-called local search or a neighborhood search improvement algorithm [1].

In the neighborhood search, as an enumeration of every node within each territory brings about a considerable change in territories and center locations, the approximate total cost in the 1-median problem might often be inaccurate. Therefore, we limited the enumeration to a certain number of nodes with lower delivery costs rather than including all nodes. Since the solution obtained by the neighborhood search is strongly dependent on its initial solution, we provide a large number of random initial locations and repeat the neighborhood search [2]. In this way, the best solution can be found from among all those. Figure 1 shows a flowchart of the random multi-start limited neighborhood search (RMLN) algorithm.

3. APPLICATIONS

We apply our model to the Tokyo metropolitan area. The following data [3] are used in these applications.

Area : Tokyo metropolitan area (Tokyo, Kanagawa, Chiba, and Saitama prefecture);

Demand nodes and candidate distribution centers : 283 nodes,
Supply nodes : 9 nodes;

Commodities : 8 (agriculture and marine, metal, machine, chemical products, etc.);

Demand, supply, and cost data : Goods flow survey in the Tokyo metropolitan area (1997);

Operating cost function : unit land price \times handled volume \times (unit required area of site + unit required floor space) \times (total demand/ handled volume)^{0.25};

Required area of site per unit handled volume :
135m²/ton \cdot day;

Required floor space per unit handled volume :
107 m²/ton \cdot day;

Average loading rate in case of transportation : 80.7% ;

Average loading rate in case of delivery : 76.4%;

Average loading capacity in case of transportation : 7.16ton;

Average loading capacity in case of delivery : 4.59ton;

Maximum number of the public distribution center : 20;

Limited number of enumerating nodes at neighborhood search: 10;

Iteration number of random initial locations : 2500000.

Using these data, we solved the mathematical programming model by the RMLN algorithm, as shown in

Figure 1, and obtained the optimal number of public distribution centers and their locations in terms of logistics costs. Figure 2 shows the transportation, distribution, operating, and total costs as the number of centers is increased. The optimal number is three, as shown in Figure 2.

Since this cost-minimizing problem were calculated on the basis of ton-kilometers, we should convert the amount of ton-kilometers by transportation and delivery to that of vehicle-kilometers using average truck loading rates and loading capacities. Consequently we can easily obtain the amount of CO₂ emissions, as the amount of CO₂ emissions discharged by trucks is considered in proportion to the sum of the vehicle-kilometers within the area. Figure 3 shows CO₂ emissions discharged by trucks within the Tokyo metropolitan area as the number of centers is increased. In this case, the optimal number of centers is four. Figure 4 shows the locations of the four centers and their territories. The difference in

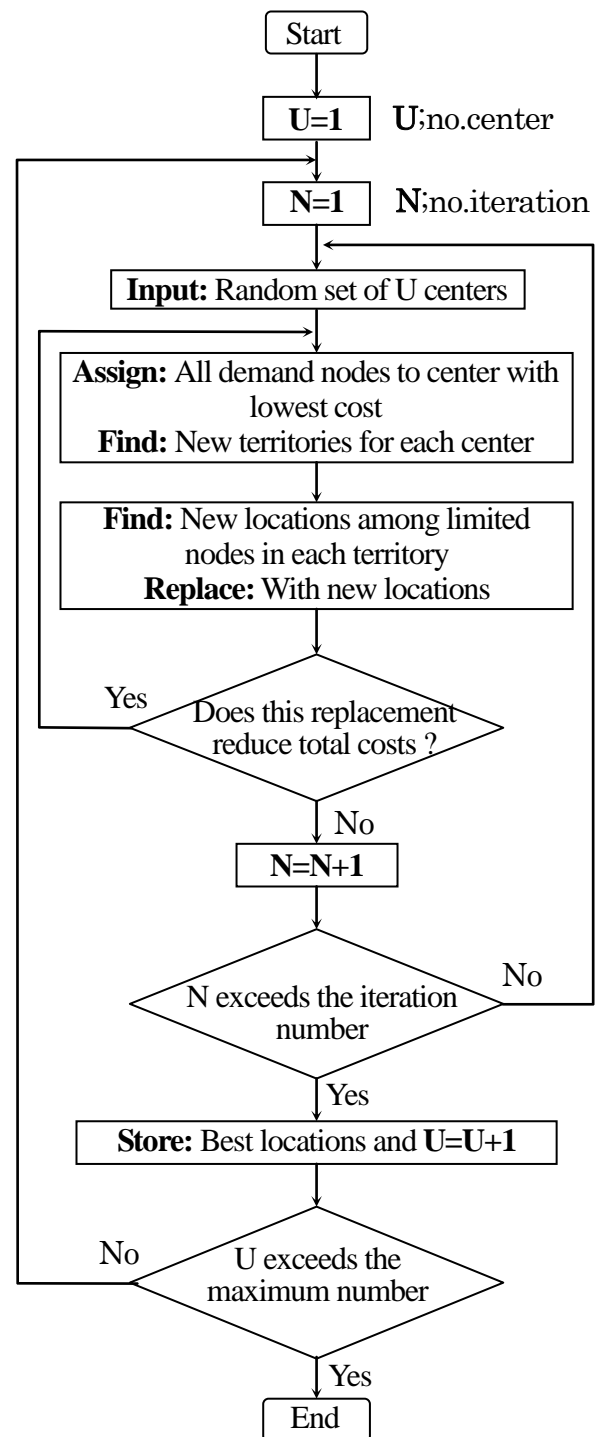


Figure 1. RMLN algorithm

the total costs between three and four locations is very small, as shown in Figure 2. From the environmental standpoint, we recommend four locations in this case.

In addition to CO₂ emitted proportional to the vehicle-kilometers, CO₂ are also discharged near the distribution centers by truck idling. If the number of centers is too small, the amount of CO₂ emissions would increase because the size of the centers would increase and traffic jams might occur near the centers. Therefore, when we take the environment into consideration, the number of centers should be increased. In our applications, having four to ten centers seems to be the best option. In that case, the logistics costs do not increase as much, as shown in Figure 2.

4.CONCLUSION

So far, we have outlined our model for the optimal location of public distribution centers and presented its applications to the Tokyo metropolitan area. Although it is difficult to reduce the amount of CO₂ emissions, we should consider the environmental issue as well as the logistics costs involved in the distribution and transportation of goods to achieve successful social logistics or “green” logistics. Public distribution centers will attempt to achieve global optimization for the community. The application of our model has enabled us to suggest an appropriate policy for public distribution centers in the Tokyo metropolitan area.

The future direction of this study will be one that incorporates the social costs for reducing the environmental contaminants into our model.

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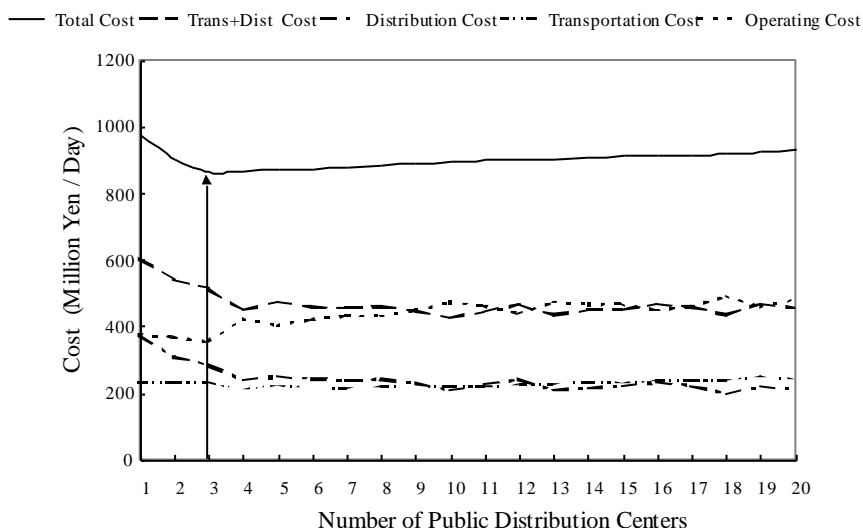


Figure2. Costs versus number of public distribution

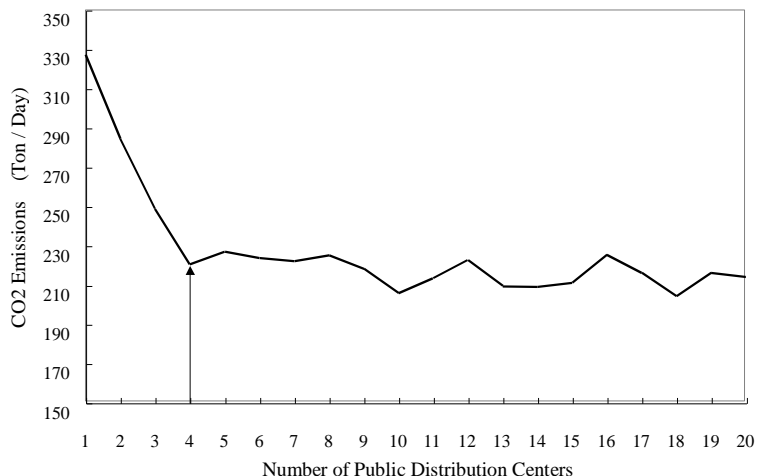


Figure 3. CO₂ Emissions versus number of public distribution centers

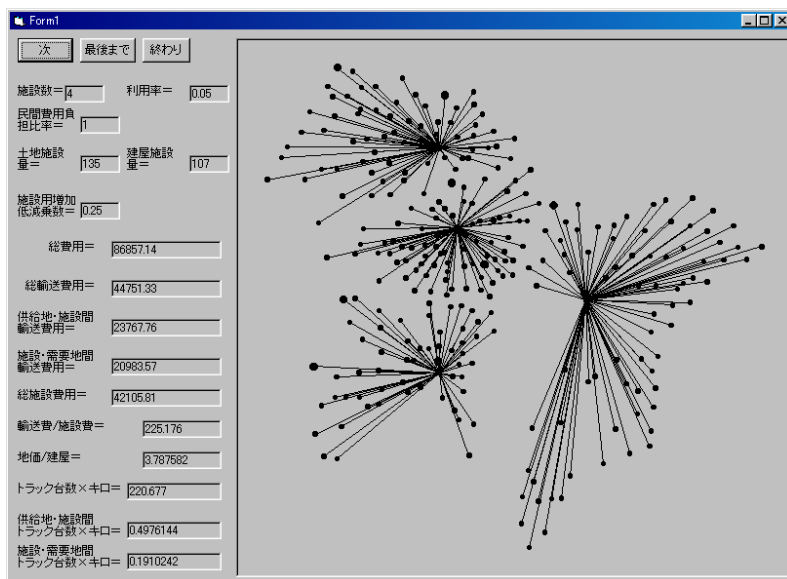


Figure 4. Locations for four public distribution centers